Comment on ”low-dimensional models for vertically falling viscous films”

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Comment on "Low-Dimensional Models for Vertically Falling Viscous Films"

In a recent Letter [1], Panga and Balakotaiah (PB) discussed the role of streamwise dissipative terms in the modeling of vertical film flows and derived a new evolution equation for the film thickness h. We show here that their equation is not free of the finite-time blowup already affecting solutions to the Benney equation [2]. As written, its relevance is therefore restricted to small Reynolds numbers. Following Ooshida [3], this equation can however be modified to avoid spurious nonphysical blowup. Unfortunately, this modification also proves insufficient to reproduce known characteristics of nonlinear waves quantitatively [3].

Using the notation and parameter definitions introduced by PB, the equation derived in [1] reads

$$\partial_t h + \partial_x \Big[h^3 + 3h^4 \partial_{xx} h + 7h^3 (\partial_x h)^2 - \frac{1}{32} \mathrm{Re} \partial_t (h^5) - \frac{27}{1120} \mathrm{Re} \partial_x (h^7) + \frac{1}{12} \mathrm{We} \mathrm{Re} h^3 \partial_{xxx} h \Big] = 0, \tag{1}$$

where Re and We are the Reynolds and Weber numbers.

The speed of one-hump solitary solutions to (1) is displayed in Fig. 1 as a function of Re for different values of the Kapitza number Ka ,which compares surface tension to viscous stress and only depends on the physical properties of the liquid considered. In each case, the turning point signals a loss of solution. This nonphysical behavior is closely connected to the occurrence of finite-time blowup in time-dependent simulations as already observed in the Benney equation [2] where blowup is promoted by the high degree of the nonlinear inertial term $\propto \mathrm{Re} \partial_{xx}(h^7)$. Ooshida [3] was able to remove the singularity in the Benney equation by a procedure akin to the Padé approximant technique. Ooshida's scalings differ from those of PB. Nevertheless, Ooshida's inertial terms can be recovered from those of PB by making use of the equivalence $\partial_t h \sim -3h^2 \partial_x h$ between time and space derivatives valid at lowest order in the long wavelength expansion. One then replaces $-\frac{1}{32} \mathrm{Re} \partial_t (h^5)$ and $-\frac{27}{1120} \mathrm{Re} \partial_x (h^7)$ in (1), by $-\frac{1}{14} \mathrm{Re} \partial_t (h^5)$ and $-\frac{27}{245} \mathrm{Re} \partial_x (h^7)$, respectively.

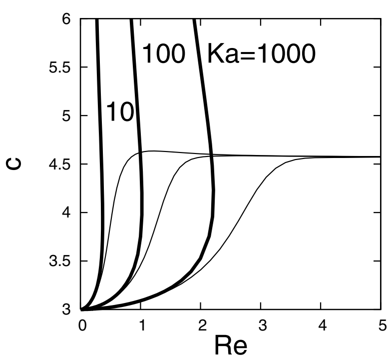

FIG. 1. Speed c of one-hump homoclinic solutions to (1) and its modification according to Ooshida's prescription [3] (thick and thin solid lines, respectively) as a function of the Reynolds number Re for different Kapitza numbers Ka.

The important result is that the modification of the inertial terms indicated above indeed suppresses the nonphysical turning back in the speed-Reynolds number relation as shown in Fig. 1. The bad news is that, as for Ooshida's improvement of the Benney equation, the speeds and amplitudes of solitary waves are much smaller than those obtained in direct numerical simulations [3].

To conclude, it seems to us hopeless to obtain a good quantitative account of the dynamics of waves in thin fluid films, even at moderate Reynolds numbers, in terms of a single evolution equation for the thickness h alone. The way out would be to recognize that the local flow rate becomes a genuine degree of freedom very early after wave inception, which would yield more elaborate models similar to Shkadov's early attempts [4].

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