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# Turbulent pattern formation in plane Couette flow: modelling and investigation of mechanisms.

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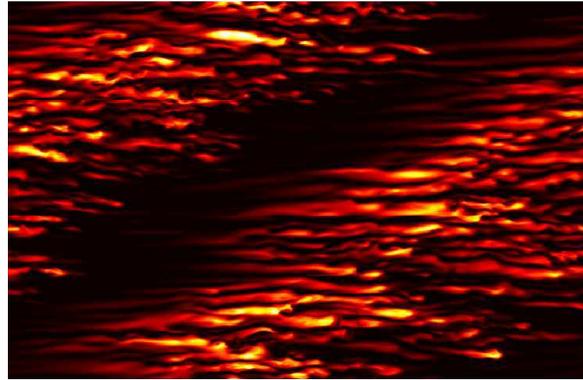
**Abstract.** In the transitional range of Reynolds number, plane Couette flow exhibits oblique turbulent bands. We focus on a Kelvin–Helmholtz instability occurring in the intermediate area between turbulent and laminar flow. The instability is characterised by means of Direct Numerical Simulations (DNS): a short wavelength instability, localised and advected in the spanwise direction. The coherent background flow on which the instability develops is extracted from DNS data, and an analytical formulation for the background flow is proposed. Linear stability analysis is performed to investigate its main mechanisms and its convective or absolute nature, depending on the location in the flow. Both DNS and linear stability analysis indicate that the instability takes place in a confined area “inside” turbulent streaks. This proceeding sums up the results from an article in preparation (Rolland, 2011).

## 1. Introduction

Plane Couette flow experiences a discontinuous transition to turbulence. Turbulence can be sustained above a first Reynolds number  $R_g$  and, provided the system size is large enough, laminar and turbulent flow can coexist: turbulence in modulated (Prigent *et al.*, 2002; Barkley & Tuckerman, 2007; Rolland & Manneville, 2011) in the form of oblique bands (figure 1) of well defined angle and wavelength. These bands can be described in terms of pattern formation (Rolland & Manneville, 2011). Mean force balance in the laminar area leads to a relation between wavelength angle and Reynolds number in good agreement with experimental findings (Barkley & Tuckerman, 2007). However, the mechanisms responsible for the formation of these bands are yet to be found.

Numerical investigation of Poiseuille pipe flow led to the identification of a Kelvin–Helmholtz instability in the trailing edge of turbulent puff and slugs (Shimizu & Kida, 2009; Duguet *et al.*, 2010). These authors discussed the development of the instability in relation with the sustainment of turbulence and the extensions of slugs.

We investigate a similar instability in the intermediate area between turbulent and laminar zones of the bands. This work will be described more thoroughly in a future paper preparation (Rolland, 2011). DNS of the flow are performed at  $R = 350$ , to identify the instability. The qualitative behaviour of the instability is identified by mean of DNS visualisation. Using time and space averages of the flow field, and insight from the visualisations, an analytical model for the background flow on which this perturbation appears is developed, the ranges of control parameters it includes are extracted from DNS. Due to lengthscale separation, local (or quasi-parallel) stability analysis (Huerre, 2000) is performed. Comparison with DNS data is made.



**Figure 1.** Norm of the velocity field, at  $y = -0.62$ .

Convectively and absolutely unstable parts of the flow are identified. Spanwise sweep is taken into account. These results are discussed in the general framework of transition to turbulence in shear flows. The finding of similar background flows in spots (Lundbladh & Johansson, 1991) suggest that some of the conclusions can be extended to spots as well.

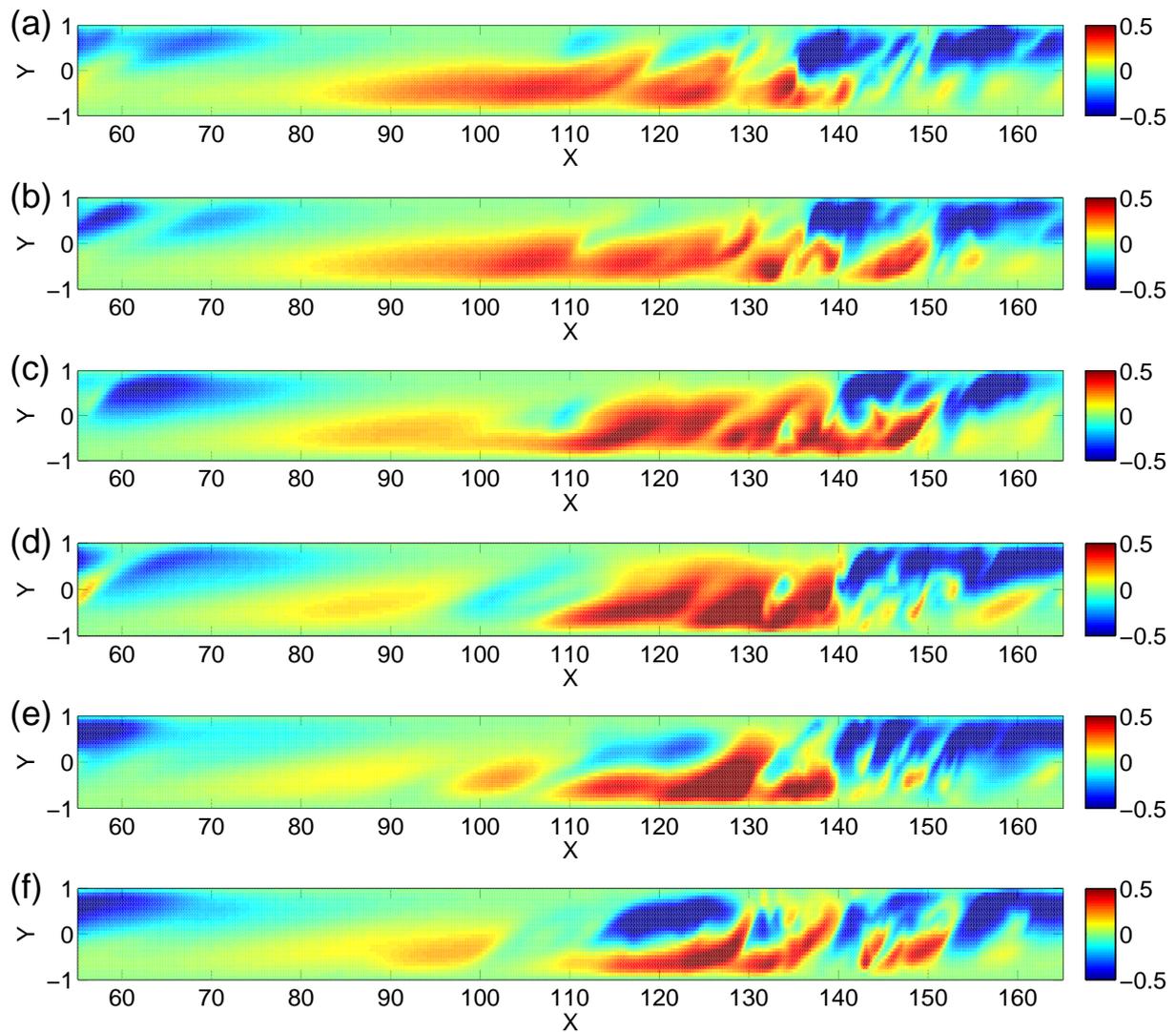
## 2. Identification

The DNS code CHANNEFLOW is used (Gibson). Lengths are made dimensionless by  $h$  the half gap between the plates, velocity by  $U$ , the speed of the counter translating plates, the Reynolds number  $R = hU/\nu$  and sizes  $L_x$  and  $L_z$  are the control parameters. In-plane periodic boundary conditions are used. A system size of  $L_x = 110$  and  $L_z = 72$  is chosen, in agreement with the optimal wavelength found experimentally and numerically (Prigent *et al.*, 2002; Barkley & Tuckerman, 2007; Rolland & Manneville, 2011). A typical flowfield can be seen in figure 1, where the square of the norm of the perturbation to the base flow  $\mathbf{v}^2$  in the  $y = -0.62$  plane is colour-coded. The so-called laminar area appears in black, the turbulent one appears in bright colours. Laminar and turbulent regions are shifted in the  $y < 0$  and  $y > 0$  parts of the flow, which leads to intermediate regions, which can be described as regions where the flow is turbulent for  $y > 0$  and laminar  $\mathbf{v} = \mathbf{0}$  for  $y < 0$  (and *vice versa*), see (Barkley & Tuckerman, 2007; Rolland & Manneville, 2011) and references therein. The area of the flow occupied by these intermediate regions can be computed, similarly to the turbulent fraction. It is non-zero as long as the bands can be educed (Rolland, 2011).

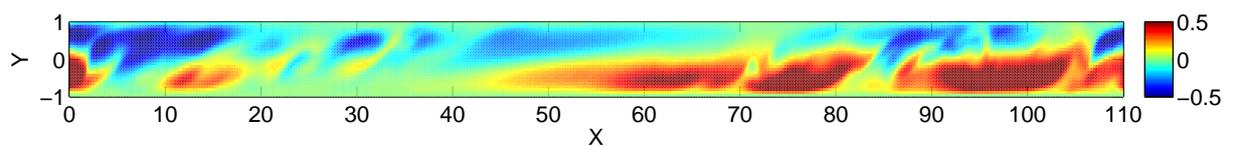
The early development of the instability is illustrated in figure 2, the streamwise component  $v_x$  of the perturbation velocity field is plotted in a given  $x - y$  plane at successive instants. One can see the perturbation appearing and developing in an intermediate area ( $95 < x < 125$ ). The shear layer and mixing of  $v_x < 0$  and  $v_x > 0$  fluid is reminiscent of a Kelvin–Helmholtz (KH) instability in the  $x - y$  plane. The activity inside the turbulent area is more complex, showing signs of both this type of KH instability and the spanwise streaks instability. The late development of the instability is illustrated on figure 3: at a different  $z$  (and different time), the roll up appears clearly around  $x = 20$ .

Examination of neighbouring positions, at  $z - 5$  and  $z + 5$  shows that the development of the instability is localised in a thin area in  $z$ , “inside” a turbulent streak, at other  $z$  positions one can see the instability at a very different stage, or no instability at all, each different occurrence of this instability is uncorrelated from the others. (Rolland, 2011)

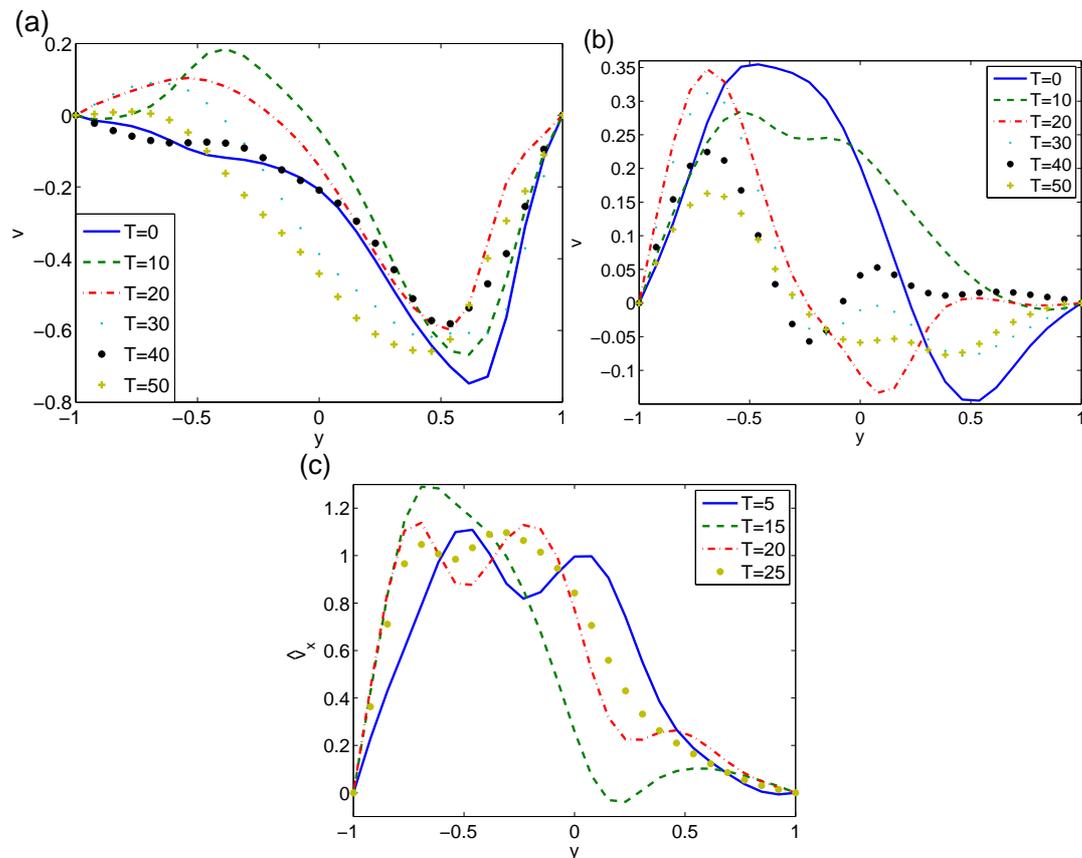
The instantaneous profiles corresponding to figure 2 can be seen on figure 4. Two positions  $x = 45$  (a) and  $x = 110$  (b) are chosen. The first one shows the shape of the velocity profile inside a turbulent streak, unperturbed by the instability. It does not change over a long



**Figure 2.** Streamwise velocity field at a given spanwise position, at successive instants. (a):  $T = 0$ . (b):  $T = 10$ . (c):  $T = 20$ . (d):  $T = 30$ . (e):  $T = 40$ . (f):  $T = 50$ .



**Figure 3.** Streamwise velocity field at a given time and spanwise position (different from that of figure 2)



**Figure 4.** Velocity profile followed in time at a given spanwise position (that of Fig. 2) for different streamwise positions: (a):  $x = 45$ , (b):  $x = 0$ . (c): wall normal dependence of the growing Fourier mode, extracted from the DNS.

period of time. The second one shows the velocity profile perturbed by the instability. Most of the perturbation is concentrated around  $y = 0$  (as seen in figure 2), while the background profile varies little. Profiles in the so-called laminar region have the expected S shape (Barkley & Tuckerman, 2007) and vary very little in time. The second intermediate area remains unperturbed, while profiles in the turbulent area fluctuate around their “S” shape (Rolland, 2011). One can follow the shape of the perturbation in time :  $x$  Fourier transform can be performed around the instability, and the shape of the normalised growing mode can be extracted at different times (figure 4 (c)). Similarly, the growth of its amplitude can be monitored Rolland (2011).

Eventually, the perturbation appear to be advected in the spanwise direction. This fact can be monitored in the Fourier transform of the velocity or vorticity fields (Rolland, 2011). The advection direction is consistent with the large scale recirculation at this location of the flow (Barkley & Tuckerman, 2007).

### 3. Model and basis linear analysis

A model background flow for the local stability analysis is developed (Rolland, 2011). Starting from the space time description initiated in (Barkley & Tuckerman, 2007), one can describe the coherent background streamwise velocity field in term of only two profiles  $v^{p,n}$ , that can be extracted from the Fourier transform of the velocity field averaged in the direction of the band.

The two profiles  $v^{p,n}$  are displayed in figure 5, where they are compared to the perturbation streamwise velocity profile averaged over the whole domain. Including the spanwise modulation of the streaks, one can write:

$$\bar{V}_x \simeq y + \bar{v}^p \alpha(z) \left( 1 + \gamma \cos \left( 2\pi \left( \frac{x}{L_x} + \frac{z}{L_z} \right) \right) \right) + \bar{v}^n \beta(z) \left( 1 + \gamma \cos \left( 2\pi \left( \frac{x}{L_x} + \frac{z}{L_z} \right) + \psi \right) \right). \quad (1)$$

The full justification of this description can be found in (Rolland, 2011), the bars corresponds to the coherent background flow. The inclusion of  $v^{p,n}$  at this scale is an approximation: the thickness of the shear layers are seen to change with  $z$  as well, for more precision one should write  $v^{p,n}(z)$ . The phase  $\psi$  appearing in this description corresponds to the shift of turbulence in the two  $y > 0$  and  $y < 0$  areas, see (Rolland & Manneville, 2011) and references therein; it is approximately  $\psi \simeq \pi/2$ . The parameter  $\gamma$  relates the average profile to  $v^{p,n}$ , one finds  $\gamma \simeq 1$ .

When describing the full velocity field, one has to include a spanwise modulation  $\alpha$  and  $\beta$ , one has  $\alpha(z) \simeq \beta(z + \lambda_z/2)$ ,  $\lambda_z$  being the wavelength of the spanwise modulation of streaks. In DNS, one perturbations are averaged, amplitude  $\alpha$  is found to be quasi-periodic (Rolland, 2011).

When investigating the background flow described by equation 1, one will be confronted with the spanwise streak instability as well as the Kelvin–Helmholtz instability we endeavour to investigate. One can use the insight given by DNS to centre the study around the relevant phenomenon: the instability develops in a thin layer in  $z$ , one can therefore make a confinement hypothesis and only study a two dimensional situation. Due to the difference in order of magnitude between the typical wavelength of the instability and the wavelength, one can make a quasi-parallel or local approximation (Huerre, 2000), using a series of profile given by:

$$\bar{V}_x = y + \underbrace{\bar{v}^p \alpha \left( 1 + \gamma \cos \left( 2\pi \left( \frac{x}{L_x} + \frac{z}{L_z} \right) \right) \right)}_a + \underbrace{\bar{v}^n \beta \left( 1 + \gamma \cos \left( 2\pi \left( \frac{x}{L_x} + \frac{z}{L_z} \right) + \psi \right) \right)}_b, \quad (2)$$

One can then scan the range of values of functions  $a \leq 0$  and  $b \leq 0$  to scan different locations of the flow: the laminar region corresponds to a small  $a = b$ , while the developing flow of  $y < 0$  intermediate region corresponds to an approximately constant  $a$  and a growing  $b$  (and vice versa for the  $y > 0$  one). one can go from one situation to another via the  $y \leftrightarrow -y$ ,  $v \leftrightarrow -v$ . However, one should be careful of the fact that for the background flow, one has to add  $z \leftrightarrow z \pm \lambda_z/2$  to recover the invariance (Rolland, 2011).

Spanwise large scale flow is included differently in the model. DNS data show that the instability is mostly sensitive to the large scale flow, contributions at small scales average to zero and have no effect on this instability. One can therefore use a dependence of the type introduced in (Barkley & Tuckerman, 2007) to study three dimensional effects and the consequence of advection on the development of the instability.

An analytical model is used to describe the background flows  $v^{p,n}$ , using two parameters, the amplitude and the inverse of the thickness of the shear layer  $d$ :

$$f^d(y) = - \left( -\tanh(d(-1 - 0.6)) + \tanh(d(y - 0.6)) \right) (1 - \exp(-0.05(1 - y))), \quad (3)$$

$$\tilde{f}^{d'} = \left( \tanh(-(1 + 0.6)d') + \tanh(-d'(y + 0.6)) \right) (1 - \exp(-0.05(1 + y))). \quad (4)$$

Using  $d$  as a parameter allows one to actually explore the variations of shear layer thickness of the flow. The maxima are located approximately at  $y = \pm 0.6$ . The streamwise coherent

background flow can be written as  $af^d(y) + bf^{d'}(-y)$ . The range of parameters found in the flow can be extracted from DNS data (Rolland, 2011),  $a$  and  $b$  vary according to the large scale modulation, one most of the time has  $d \neq d'$ . Fluctuations of  $a$ ,  $b$ ,  $d$  and  $d'$  are found to be of the same order of magnitude as their average: this is partly caused by the spanwise modulation of streaks. They are used as parameters for the local stability analysis. The formulation of equation 2 and the sampled values (Rolland, 2011) then serve as guideline as to whether a regime of parameter is likely to occur and which region of the flow they correspond to. A comparison to  $v^{p,n}$  can be seen in figure 5 (a). Investigating the stability of the coherent background flow then corresponds to exploring a subset of parameter space  $a, b, d, d'$ , keeping in mind the organisation of the flow. Similarly, the analytical expression of  $f$  can be used to describe the spanwise large scale flow (Rolland, 2011).

Exploration of the space parameters shows that the unstable profiles correspond to those whose amplitude and inverse of shear layer thickness are large enough, that correspond to those found “inside” turbulent streaks. This is in agreement with what is found in the direct numerical simulations.

The stability analysis is performed on the Orr–Sommerfeld equation, and linear analysis of Orr–Sommerfeld–Squire system is additionally performed to rule out non-trivial three-dimensional effects. Wall normal dependence is approximated by two basis of functions fitting the boundary conditions (Rolland, 2011), a Galerkin truncation procedure is applied to obtain the eigen value problem.

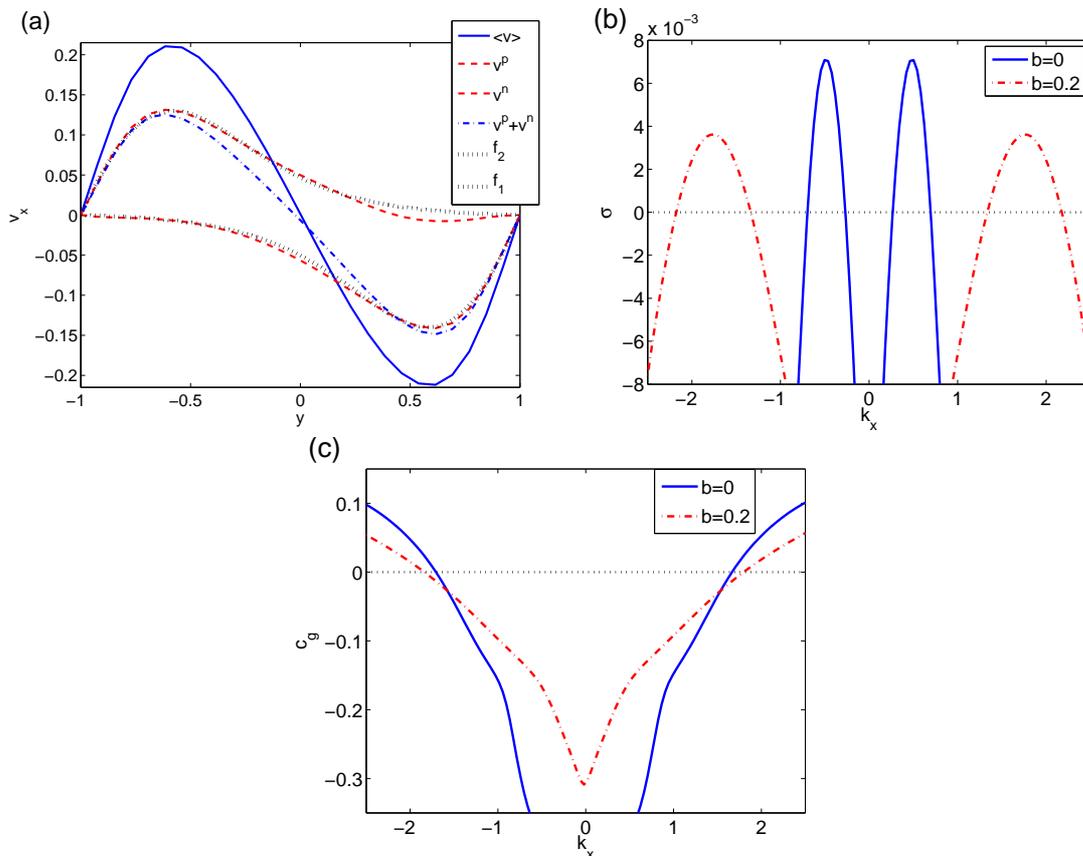
#### 4. Results of linear stability analysis

One can first consider a few typical test cases to illustrate the main characteristic of the linear analysis. The main conclusions of such a study are confirmed by a full parametric study (Rolland, 2011). The range of  $300 \lesssim R \lesssim 600$  for which bands and low turbulence spots are seen is considered. Two cases are presented here, at  $R = 500$ , one at  $a = 0.7$  and  $b = 0$ , corresponding to the heart of the intermediate region, and another at  $a = 0.7$  and  $b = 0.2$ , corresponding to a situation between intermediate and turbulent. The parameter  $b$  continuously increases the most unstable wavevector (figure 5 b). The values of  $k$  found in DNS, correspond to moderate values of  $b \neq 0$ . However, parameter  $b$  has little qualitative effect on the group velocity  $c_x$  (figure 5 c) the amplitude of  $c_x$  is changed, the (real) wavevector for which  $c_x$  crosses 0 remain approximately the same. The main difference between these two cases lie in the growth rate corresponding to  $c_x = 0$ , the  $b \neq 0$  case can be absolutely unstable, whereas the  $b = 0$  one is convectively unstable. In some cases  $b \neq 0$  can also stabilise the flow. The growth rate  $\sigma$  is unchanged by application of the  $y \leftrightarrow -y$ ,  $\bar{V} \leftrightarrow -\bar{V}$  symmetry, while the group velocity changes sign. When the flow is convectively unstable, the sign of the group velocity  $c_x$  always corresponds to an advection of the perturbations toward the turbulent area  $c_x > 0$  if the intermediate area corresponds to activity in the  $y < 0$  part of the flow, and vice versa.

Three dimensional stability analysis of the Orr–Sommerfeld–Squire equations shows that there are no non trivial three dimensional effects. The sole effect of the spanwise background flow is advection of the perturbation: computation of the  $c_z$  group velocity for a range of profiles corresponding to the large scale flow show that it typically crosses zero in the core of the turbulent area. Similarly, wavevectors  $k_x, k_z$  of  $c_z = 0$  and  $\sigma > 0$  are found for background flow corresponding to the turbulent region, while the typical intermediate background flow are just convectively unstable.

#### 5. Concluding remarks

Identification of small scale instability is one of the advantages of DNS over experiment. It was reported in Poiseuille pipe flow, see Duguet *et al.* (2010); Shimizu & Kida (2009). In the case



**Figure 5.** (a): Average velocity profile (blue full line) and profile of the modulation (red dashed lines), extracted from DNS data, compared to the model background flow (black dotted line). (b,c): Example of results of the linear analysis: (b): growth rate, (c): group velocity for a regime of parameter consistent with DNS data ( $R = 500$ , amplitude  $a = 0.7$ , inverse of shear layer thickness  $d = 3.9$  for two value of amplitude  $b$ ).

of plane Couette flow, one can use DNS data to construct a model for linear stability analysis. The stability analysis helps one understand the characteristics of this Kelvin–Helmholtz type of instability, however its main interest is to emphasise the convective/absolute transition occurring in the flow ((Huerre, 2000)) as one goes from the turbulent intermediate area to the turbulent one. The sign of the  $c_x$  group velocity always corresponds to transport of the perturbation toward the turbulent area. Inclusion of the spanwise background flow indicates that the absolutely unstable nature of the turbulent region is not changed, however, the perturbations are then convected alongside the bands.

The parallel between an absolute/convective transition and triggering of turbulence has often been made (see (Huerre, 2000) and references therein). A similar argument was proposed in the case of Poiseuille pipe flow: (Shimizu & Kida, 2009) considered the advection of the perturbations toward the turbulent region, (Duguet *et al.*, 2010) compared the advection speed of the vortices to that of the turbulent region for different Reynolds number (and corresponding regimes). However PPF is less adapted to such a stability analysis, the unstable base flow moves with the “fronts”, there exist no frame in which this background flow is at rest. It was argued by some of these authors Shimizu & Kida (2009) that the sustaining of turbulence could be expressed as a self sustaining cycle. A similar argument can be proposed for plane Couette flow (Rolland,

2011): Turbulence generates a background flow that is unstable, that instability then feeds back on turbulence.

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