Reply to "Comments on 'A Density Current Parameterization Coupled with Emanuel's Convection Scheme. Part I: The Models'"

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1. Introduction

In the original paper of this discussion, Grandpeix and Lafore (2010, hereafter GL10) presented a density current parameterization coupled with Emanuel's convection scheme. Jun-Ichi Yano's comment (Yano 2012, hereafter Yc12) questions various formulas of this parameterization and proposes new ones following an approach based on segmentally constant approximation (SCA; Yano et al. 2010). Essentially, Yc12 ascertains an overall agreement between the new formulas and the GL10 ones, the proposed modifications amounting to the introduction of new terms representing various processes neglected in GL10. Yet Yc12 claims that it was a mistake not to use SCA for the wake equation derivation. Although we acknowledge the interest of the new developments presented by Yc12, we think that the very existence of two different ways of developing wake models should be considered as very positive. Let us emphasize, however, that most of the equations put forward by Yc12 have never been published and appear for the first time in this comment: what we are dealing with here is not really a comment but rather a note about new developments of SCA.

Our reply to Yc12 is structured in two parts: 1) a first part devoted to a global analysis of Yc12 and of its relevance for the wake model of GL10, and 2) a second part (hereafter referred to as the appendix) that contains detailed item-by-item responses to Yano's remarks.

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In what follows, we shall keep the equation numbering proposed in Yc12: all of the equations numbered without a prefix are those from GL10, unless otherwise noted. The equations in Yc12's main text and appendix are numbered with prefixes 2 and A, respectively.

2. General remarks

a. The modifications proposed in Yc12

The first correction proposed by Yc12 consists of taking into account the variation of the fractional area of wakes σ_w with altitude (item 2 of the appendix). We think that since wakes are shallow objects (typically 1 km in height and tens to hundreds of kilometers in diameter), this generalization is not relevant for their representation. We shall come back to this generalization when we consider the claim by Yc12 that the wake parameterization may be considered as a generalization of the convective parameterization by Arakawa and Schubert (1974, hereafter AS74).

The second correction consists of introducing terms representing the effect of the propagation of wakes from one grid cell to the other (item 4 of the appendix). This generalization is worth considering and sketches a way for future developments; however, more work is necessary before it becomes usable. For instance, unproven formulas are used in the mathematical derivation of the main equations. Anyway, this direction of research appears promising.

b. Wake models: SCA and statistical approaches

Yc12's approach is based on the idea that the wake equations may be derived within the framework of SCA (item 1 of the appendix). Such an approach is very far from ours, which is more statistical in nature (see again item 4 of the appendix) and does not attempt to follow the evolution of each object (wake or convective tower). It is akin to the idea that parameterizations should result from some truncation applied to cloud-resolving models (CRMs). As a matter of fact, SCA should yield, after some more developments, a CRM, from which it is hoped that, by reducing the number of segments, a parameterization will emerge. While this line of development is interesting and should be pursued, our statistical/probabilistic way is also worth considering. Up to now it has been the most frequently used (e.g., in AS74) and its ability to provide parameterizations is established.

Then the question arises of whether SCA can be applied to a statistical representation of segmentally constant objects. As mentioned in items 4 and 6 in the appendix, this is not obvious at all.

c. The difficulty of using SCA

Another point that makes it difficult to use SCA for wake parameterization is the lack of maturity of SCA. Indeed, the sole implementation of SCA we know of is the one presented in Yano et al. (2010), a paper published after GL10. Moreover, this implementation is only 1D in the horizontal direction; the equations for a 2D horizontal domain are presented for the first time in Yc12 with some incomplete proofs (see item 4 in the appendix). Thus, again, even though the development of a parameterization from SCA principles is a very interesting matter, the statistical/probabilistic approach is certainly more promising in the short term.

Let us emphasize another aspect of Yc12's approach that does not appear relevant to us, namely that the wake parameterization is similar to a convective mass flux parameterization. Yc12 emphasizes the analogy between GL10's formulas and those of AS74. But this analogy is far from exact. On the contrary, the comparison between the two sets of equations puts forward the fundamental differences between the two parameterizations: 1) in the convective case (AS74), the fractional area covered by convective drafts is much smaller than unity, while in the wake case (GL10) it is on the order of unity; 2) the AS74 convective parameterization is a mass flux parameterization while the GL10 wake parameterization is not, which manifests itself by the presence of a prognostic equation for σ_w; and 3) consistent with points 1 and 2, the AS74 equations are in flux form while GL10's equations are in advective form.

d. Convective updraft and SCA

In GL10 the coupling between the wake scheme and the convection scheme is performed thanks to the available lifting power (ALP) variable. To use this variable, a budget equation is derived for the kinetic energy in a convective updraft; it allows us to relate the power available at cloud base (the ALP) to the mass flux in the updraft, thus providing a closure for the convective scheme.

Yc12 attempts to derive a similar relation from SCA. His Eq. (2.8) agrees with our Eq. (31) when turbulence is neglected (i.e., when $\langle w''^2 \rangle = 0$). But the turbulence terms in the two relations have different coefficients. We believe (see item 15 in the appendix) that this difference comes from an ill account of the turbulence in SCA equations.

3. Concluding remarks

- The statistical approach that we chose is relevant and yields a wake model that is correct from the statistical point of view as well as from the SCA point of view.
- SCA may presumably be used in order to derive the wake equations; however, the derivations need be established on sounder grounds.
- Taking into consideration the slope of the wake boundaries does not appear to be relevant modification for the wake model.
- The other developments proposed by Yc12 open an encouraging way for dealing with the propagation of wakes from one grid box to another.
- Paradoxically, it is in the representation of convective updrafts that SCA appears to fail. This is due to the neglect of the effect of turbulence within updrafts, an approximation quite questionable in the case of convective updrafts.

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APPENDIX

Detailed Analysis of J.-Y. Yano's Comment

In this appendix, we list our responses to J.-I. Yano's comments in the order in which they appear in Yc12 (the numbering is ours). J.-I. Yano's quoted text is italicized.

a. Item 1

The wake parameterization proposed by Grandpeix and Lafore (2010, hereafter GL10) can be considered as a special application of the general mode decomposition approach proposed by Yano et al. (2005). According to *the latter work, the mass flux–based parameterization, originally introduced by Arakawa and Schubert (1974, hereafter AS74), can be generalized by considering it as an approach based on segmentally constant mode decomposition [or segmentally constant approximation (SCA); Yano et al. 2010]. The idea of SCA consists of subdividing a gridbox domain into a number of constant-value segments in different sizes and shapes, each representing various subgrid-scale subcomponents, not only the convective elements but also such elements as the wake. In this respect, GL10's wake parameterization is a particular case of generalization of the mass flux convection parameterization based on SCA.*

Generally speaking, it is true that the GL10 wake parameterization is based on a decomposition of the domain in subelements where state variables are horizontally homogeneous. In that sense, it can be considered as a special case of SCA. However, contrary to what is asserted by Yc12, the wake elements are not necessarily subgrid scale (GL10, p. 884). In that respect, the GL10 wake parameterization is not a generalization of the mass flux convection parameterization based on SCA; indeed, as mentioned in section 2, it is not a mass flux parameterization.

Obviously, this is not the point of view of Yc12, who adds:

Unfortunately, GL10 developed their parameterization on their own without referring to either Yano et al. (2005) or AS74. This led GL10 to various mistakes and ambiguous assumptions. This paper lists corrections based on rigorous application of SCA. The list intends to demonstrate the close link of their parameterization to the mass flux formulation as well as the importance of developing a parameterization from first principles as emphasized by Yano et al. (2005).

Admittedly, we should have referred to AS74, since they developed their parameterization along lines similar to ours; we did not think of it because the actual application field is quite distinct from ours. Referring to Yano et al. (2005), on the contrary, does not appear relevant since they were mainly concerned with fixed boundary subcomponents and with representation of mass flux components in wavelet space.

In spite of that, Yc12 considers as obvious that the GL10 wake parameterization should follow the SCA principles. From this it follows that all deviations from these principles are called ‘‘mistakes’’ or ‘‘ambiguous assumptions’’ or ‘‘errors.’’ We shall consider instead that we are faced with two distinct ways of designing parameterizations, which are compatible in some respects and incompatible in others. Thus, ‘‘mistakes,’’ ‘‘errors,’’ and other similar terms should be simply understood as ‘‘differences from SCA principles.’’

b. Item 2

Equation (7), presented without derivation, should read

$$\partial_t S_w = -\partial_p \int_{\Sigma_w} \omega_w \, d\Sigma + \int_{\Gamma_{w,\mathrm{in}}} (\mathbf{V}^* - \mathbf{V}_\Gamma) \cdot \mathbf{n}_w \, d\Gamma + \int_{\Gamma'_w} \mathbf{V} \cdot \mathbf{n}' \, d\Gamma, \quad (2.1a)$$

$$\partial_t S_x = -\partial_p \int_{\Sigma_x} \omega_x \, d\Sigma - \int_{\Gamma_{w,\mathrm{in}}} (\mathbf{V}^* - \mathbf{V}_\Gamma) \cdot \mathbf{n}_w \, d\Gamma + \int_{\Gamma'_x} \mathbf{V} \cdot \mathbf{n}' \, d\Gamma, \quad (2.1b)$$

where

$$\mathbf{V}^* = \mathbf{V} - \omega \partial_p (\mathbf{r}_{\Gamma,w}), \quad (2.2)$$

with $\mathbf{r}_{\Gamma,w}$ *designating the position of the wake boundary. This is obtained directly from Eq. (A.6) by setting* $j = w$ *and* x*. Their Eq. (7) is found only if the wake boundaries are perfectly vertical.*

The original equation [Eq. (7)] differs from Eqs. (2.1a), (2.1b), and (2.2) by the differentiation ∂_p being within the integral in the integration terms over Σ_w and Σ_x and by the replacement of $\mathbf{V}^*$ by $\mathbf{V}$ in the integrals over the wake contours. Thus the two equations are identical when the wake boundaries are supposed independent of the vertical coordinate.

Yc12's short paragraph implicitly suggests three important statements.

The first statement is that, if one assumes, as is done in GL10, that the wake boundaries are vertical, then Eq. (7) is correct; this agreement between the two approaches might be considered as a verification of both our formula and SCA.

The second statement is more explicit. It states that if one relaxes this assumption, then Eqs. (2.1a), (2.1b), and (2.2) are the right formulas. We agree that Eqs. (2.1a), (2.1b), and (2.2) are correct. However, we would like to emphasize that implementing such a complete representation of the evolution of the wake contour vertical profiles requires a full description of the velocity field, which may be quite difficult. As a matter of fact, it is impossible in GL10's model, since the dynamic fields are only represented by the prescription of the vertical profile of the vertical velocity difference $\delta\omega$. As concerns SCA, the equations necessary for such an implementation have not yet been published. Especially, Eq. (3.4) of Yano et al. (2010), which is presented in Yc12's appendix as ‘‘a two-dimensional case with a moving subcomponent boundary'' does not include any term representing the subcomponent boundary movement. Moreover, only subcomponents with vertical walls are represented in Yano et al. (2010), with the variation of cross section with altitude coming only from the finite vertical extension of each subcomponent. Hence the proposed complication is at most a way for future developments; it is certainly not an improvement to be implemented presently.

The last statement is not as explicit but is recurrent all along Yc12. It states that the hypothesis that the wake boundaries are vertical is not a relevant hypothesis. We disagree with this statement because wakes are shallow objects: a typical value of the spread velocity of wakes is 10 m s^{-1}, so that wakes are more than 80 km large after 1-h lifetime, which is comparable to a grid cell size, while their height is typically less than 2 km. For objects with such a small aspect ratio, the nonverticality of the boundaries is expected to be unimportant. This is not the case, of course, for the objects for which SCA was designed, namely cumulus and convective towers. Because of this important difference we think that referring to AS74 is not relevant for the wake parameterization problem.

c. Item 3

The flag η is introduced below Eq. (7): it is against the principle of scale separation to consider the case in which all wakes are clustered around the center of a grid box so that they do not cross the gridbox boundary. It is not clear how wakes can physically know the existence of a gridbox boundary.

The η parameter was introduced in order to deal with special cases. When using the wake parameterization in a GCM, η is set to 1 (the nonconfined case) since cold pools cannot know the existence of gridbox boundaries. However, the experimental design may be such that the simulated domain includes entirely the convective system. Such is the case for the Global Energy and Water Cycle Experiment (GEWEX) Cloud System Study (GCSS) Case 1 [a squall line case during the Tropical Ocean and Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE)] where the domain moves with the squall line. In such a situation the η parameter is set to 0.

d. Item 4

Equation (8) should read as follows:

$$\int_{\Gamma'_w} \mathbf{V}\cdot\mathbf{n}'\,d\Gamma = S_t\nabla\cdot\sigma_w\overline{\mathbf{V}}_w. \qquad (2.3)$$

Here, $\overline{\mathbf{V}}_w$ is the velocity averaged over the wake regions, which is a slowly varying function over the grid boxes. The result is obtained by setting $j = w, \phi_j = 1$ in Eq. (A.5). The original Eq. (8) is found only if we assume that (i) $\overline{\mathbf{V}}_w = \overline{\mathbf{V}}$ and (ii) σ_w is constant and independent of large-scale (gridscale) coordinates.

In GL10 no attempt was made to represent the proper movement of wakes or the propagation of wakes from one grid cell to another. As a consequence the wake field is considered as homogeneous in space: the sole effect of large-scale flow is through large-scale convergence. Yc12 agrees with our formulation in the case where wakes have no proper movement and the wake field is homogeneous. His formulation goes a step further and yields a way to account for wake proper movement and propagation. To that end two new fields σ_w and $\overline{\mathbf{V}}_w$ are introduced, representing local wake fraction and velocity, respectively. This is an interesting proposition. However, the derivation of Eq. (A.5) [from which Eq. (2.3) is derived] given in Eq. (A.2) is based on a crudely discretized formulation of the integral on the left-hand side of Eq. (2.3) and is not very convincing. Furthermore, the derivation of Eq. (A.5) begins with an equation relating the length of a gridbox side $\Delta\Gamma_w$ that lies within wakes with the surface fraction σ_w:

$$\Delta\Gamma_w = \sigma_w L,$$

where L is the length of the gridbox side, which is true in the statistical sense (it can be proved using a computation similar to the one in appendix A of GL10) but wrong for specific wake configurations. So the SCA derivation of Eq. (A.5) has still to be found. Finally, the very definition of the continuously differentiable fields σ_w and $\overline{\mathbf{V}}_w$ is not clearly stated. Thus some more work is necessary before this proposition is usable.

e. Item 5

In their definitions of the entrainment and detrainments rates, e_w and d_w, given just before Eq. (10), $\mathbf{V}$ should be replaced by $\mathbf{V}^*$ [cf. Eq. (2.2)]. Note that this is a direct consequence of a mistake found in their Eq. (7).

This is not a mistake but a consequence of the hypothesis that wake edges are considered vertical in GL10: see item 2.

f. Item 6

Equation (10) should read

$$\partial_t\sigma_w = -\partial_p(\sigma_w\overline{\omega}_w) + e_w - d_w - \nabla\cdot(\sigma_w\overline{\mathbf{V}}_w) \qquad (2.4)$$

as derived in section C of the appendix. This equation is comparable to Eqs. (43) and (47) of AS74. Note that in AS74, the possibility of convective elements crossing the gridbox boundary is excluded, so the last term in Eq. (2.4) is missing. Both Eqs. (11) and (12) must also be modified accordingly.

We agree that Eq. (2.4) is correct and accounts for the effects induced by the heterogeneity of the wake field among the grid boxes (see item 4). However, it is not a generalization of Eq. (10), since it is not expressed in term of $\delta\omega$ but in term of $\overline{\omega}_w$. Equation (2.4) should rather be considered as the SCA equivalent of Eq. (10).

If one wants to obtain a generalization of Eq. (10), then one has to get back to the very definitions of the fields σ_w and $\overline{\mathbf{V}}_w$ and to their role in the entrainment and detrainment terms. Hence, again, some more developments are necessary in order to get a full set of equations accounting for the wake movements across the grid boxes.

g. Item 7

We should set $\delta\omega^{\mathrm{cv}} = -g[M_p/\sigma_w - (M_c + M_e)/(1 - \sigma_w)]$ rather than $\delta\omega^{\mathrm{cv}} = -g[M_p/\sigma_w - M_c/(1 - \sigma_w)]$ because the nonwake area is occupied by both the convection and the environmental subcomponents.

This sentence is irrelevant: $\delta\omega^{\mathrm{cv}}$ is used only at upper levels where the environment of convection is homogeneous between wake and off-wake regions; the ''environmental subcomponents'' are present as well in the wake area as in the off-wake area.

h. Item 8

Equation (13) should read

$$\partial_t\left(\int_{\Sigma_w}\theta_w\,d\Sigma\right) = \int_{\Sigma_w}\frac{\theta_w}{T_w}\frac{H_w}{C_p}d\Sigma - \partial_p\int_{\Sigma_w}(\omega_w\theta_w)\,d\Sigma + \int_{\Gamma_{w,\mathrm{in}}}(\mathbf{V}^* - \mathbf{V}_\Gamma)\cdot\mathbf{n}_w\theta\,d\Gamma + \int_{\Gamma'_w}\mathbf{V}\cdot\mathbf{n}'\theta\,d\Gamma, \tag{2.5a}$$

$$\partial_t\left(\int_{\Sigma_x}\theta_x\,d\Sigma\right) = \int_{\Sigma_x}\frac{\theta_x}{T_x}\frac{H_x}{C_p}d\Sigma - \partial_p\int_{\Sigma_x}(\omega_x\theta_x)\,d\Sigma - \int_{\Gamma_{w,\mathrm{in}}}(\mathbf{V}^* - \mathbf{V}_\Gamma)\cdot\mathbf{n}_w\theta\,d\Gamma + \int_{\Gamma'_x}\mathbf{V}\cdot\mathbf{n}'\theta\,d\Gamma \tag{2.5b}$$

as a special case of Eq. (A.4). These equations are a generalization of Eqs. (14), (15), (44)–(46), and (48)–(50) of AS74.

This change of the equation is again a consequence of the relaxation of the vertical wall hypothesis. See item 2.

i. Item 9

Regarding assumption (ii) immediately below Eq. (14), it is wrong to replace θ and $\mathbf{V}$, respectively, by $\overline{\theta}_j$ and $\overline{\mathbf{V}}_j$, averaged over the given fixed grid box with $j = w$ and x in the contour integrals over Γ'_j. They even fail to mention the assumption concerning $\mathbf{V}$. It is a mistake to neglect the fact that all of the subgrid-scale variables, including $\overline{\theta}_j$ and $\overline{\mathbf{V}}_j$, are varying over the large scale (gridscale). This mistake culminates as the disappearance of the large-scale (gridscale) horizontal advection in their final result, Eq. (19).

Let us recall that assumption (ii) immediately below Eq. (14) states that the average value of θ over the part of the gridbox boundary that lies within (outside) wakes is $\overline{\theta}_w$ ($\overline{\theta}_x$). This assumption is consistent with GL10's hypothesis of a statistically spatially homogeneous field of wakes. It does not account, of course, for large-scale variations of the wake field.

Hence, this item does not bring anything new. It is merely a restatement that the model should represent the wake proper movements and propagation across grid boxes.

j. Item 10

Equation (16) should read

$$\sigma_w\partial_t\overline{\theta}_w = \frac{1}{S_t}\int_{\Sigma_w}\frac{\theta_w}{T_w}\frac{H_w}{C_p}d\Sigma - \partial_p\left[\sigma_w\overline{(\omega''_w\theta''_w)}\right] - \sigma_w\overline{\omega}_w\partial_p\overline{\theta}_w - e_w\delta\theta - \sigma_w\overline{\mathbf{V}}_w\cdot\nabla\overline{\theta}_w, \tag{2.6a}$$

$$(1-\sigma_w)\partial_t\overline{\theta}_x = \frac{1}{S_t}\int_{\Sigma_x}\frac{\theta_x}{T_x}\frac{H_x}{C_p}d\Sigma - \partial_p\left[(1-\sigma_w)\overline{(\omega''_x\theta''_x)}\right] - (1-\sigma_w)\overline{\omega}_x\partial_p\overline{\theta}_x + d_w\delta\theta - (1-\sigma_w)\overline{\mathbf{V}}_x\cdot\nabla\overline{\theta}_x. \tag{2.6b}$$

Note that the advection by wake and off-wake flows is included in Eqs. (2.6a) and (2.6b) as a result of the correction.

This change of the equations is again a consequence of the relaxation of the vertical wall hypothesis (see item 2) and of the accounting for the wake proper movement and propagation (see item 4).

k. Item 11

If the paired Eqs. (18) are derived correctly, the first of the pair should correspond to Eqs. (44) and (48) of AS74, and the second to Eq. (16) of AS74, when the static energy in the latter is redefined as potential temperature. However, note that $\eta = 0$ is implicitly assumed in AS74.

Equations (18) ''correspond to Eqs. (44) and (48) of AS74'' only in a very loose sense. Admittedly, the two sets of equations express the evolution of dry static energies or, equivalently, of potential temperatures as sums of a diabatic heating term, an entrainment/detrainment term, and a vertical mass flux term. However, GL10's equations are in advective form, whereas those in AS74 are in flux form. This difference stems from the fact that the two sets deal with very different objects: AS74 are dealing with convective-scale drafts, whose horizontal dimensions are negligible when compared to the gridbox size, whereas GL10 are dealing with wakes, whose horizontal dimensions are comparable to the gridbox size. This matter is already discussed in item 2.

l. Item 12

Equation (19) should read

$$\partial_t\overline{\theta} = -\overline{\omega}\partial_p\overline{\theta} - [\sigma_w\overline{\mathbf{V}}_w\cdot\nabla\overline{\varphi}_w + (1-\sigma_w)\overline{\mathbf{V}}_x\cdot\nabla\overline{\varphi}_x] + \frac{Q_1' + Q_1^{\mathrm{wk}}}{C_p}, \tag{2.7a}$$

$$\partial_t\delta\theta = -\overline{\omega}\partial_p\delta\theta - (\overline{\mathbf{V}}_w\cdot\nabla\overline{\varphi}_w - \overline{\mathbf{V}}_x\cdot\nabla\overline{\varphi}_x) + \frac{\delta Q_1' + \delta Q_1^{\mathrm{wk}}}{C_p}. \tag{2.7b}$$

after corrections.

The corrections brought in by Yc12 are (i) the addition of large-scale advection terms, which is again the representation of the effect of wake movement from one grid box to another, and (ii) the suppression of the damping term by gravity waves. We strongly disagree with this suppression as is explained in the response to the next item.

m. Item 13

There is no possible way for obtaining the so-called gravity-wave radiation term in the second of the paired Eqs. (19) when assumption (iii) immediately below Eq. (14), corresponding to the upstream approximation, is strictly applied.

Let us recall that assumption (iii) in GL10 reads ''the average value of θ over Γ_{in}^{+} (i.e., over the part of the wake contour where the wakes entrain) is $\overline{\theta}_x$ and the average value of θ over Γ_{in}^{-} (i.e., over the part of the wake contour where the wakes detrain) is $\overline{\theta}_w$; it corresponds to the upstream approximation.''

We agree with Yc12's statement, since we never derived the GW term from assumption (iii).

As said on page 888 just after Eqs. (19) ''where we have added a supplementary term to account for the mass adjustment by gravity waves (GWs).'' This formulation is not derived but is a heuristic representation of the effect of GWs as explained in section 2a (''Assumptions and conceptual model'') at page 884 when we said ''The temperature difference between the wake and the off-wake regions is reduced by the mass adjustment by gravity waves (GWs); this damping process is zero at the surface, where gravity waves cannot occur, which permits the existence of wakes. Above the surface, it grows with altitude.'' The details of the GW damping time scale are provided at page 889 (item v) based on the GW linear theory.

Jun-Ichi Yano makes the following proposition: *An obvious way for getting the gravity-wave radiation term is to assume instead that the wake boundary value for θ does not strictly follow an upstream approximation, but rather represents a finite deviation from this state designated by θ_Γ''. As a result, we find an additional term*

$$\frac{1}{\sigma_j S_t}\int_{\Gamma,j}(\mathbf{V}^* - \mathbf{V}_\Gamma)\cdot\mathbf{n}_j\theta_\Gamma''\,d\Gamma \tag{2.8}$$

on the right-hand side of both Eqs. (18) with $j = w$ and x, respectively, and $\sigma_x = 1 - \sigma_w$. A closed expression for this term is obtained by applying the eddy–diffusion hypothesis introduced by, for instance, Asai and Kasahara (1967) with an eddy–diffusion coefficient given by μ_e. Then we add the terms $-(\mu_e/\sigma_w)\delta\theta$ and $\mu_e/(1-\sigma_w)\delta\theta$, respectively, to the right-hand side of the first and the second parts of Eqs. (18). This furthermore adds a term $-\mu_e[1/\sigma_w + 1/(1-\sigma_w)]\delta\theta$ to the right-hand side of the second in the pair of Eqs. (19), or Eq. (2.7b). GL10's original expression is found by setting $k_{GW}/\tau_{GW} = \mu_e[1/\sigma_w + 1/(1-\sigma_w)]$.

And he concludes as follows: *As this derivation suggests, this damping term has nothing particularly to do with the gravity waves but is simply a sum of all transport by ''eddies'' (including turbulent mixing) crossing the wake boundary… We should also realize that addition of this term is inconsequential because this is simply equivalent to changing the entrainment and detrainment rates from e_w and d_w to $e_w + \mu_e$ and $d_w + \mu_e$, respectively.*

We disagree with this point of view. Gravity waves are a nonlocal process, involving a pressure field that is a nonlocal variable. GWs have nothing to do with turbulence mixing, which is a much more local process. For instance, GWs allow a fast mass adjustment far from the source. Such a process is impossible with the turbulent mixing. In our parameterization we do not treat pressure field so that we cannot account for the effect of GWs on the mass field. Hence the GW term has been added and we performed a sensitivity study in Part II (Grandpeix et al. 2010) of our series of papers.

n. Item 14

The first of the paired Eqs. (20) should read

$$\frac{Q_1^{\mathrm{wk}}}{C_p} = -[\partial_t \sigma_w + (e_w - d_w)]\delta\theta - \delta\omega\sigma_w(1 - \sigma_w)\partial_p\delta\theta \tag{2.9}$$

by correctly performing the derivation of Eqs. (19) from Eqs. (17) and (18).

The original formula [Eq. (20)] reads as follows:

$$\frac{Q_1^{\mathrm{wk}}}{C_p} = +[\partial_t \sigma_w - (e_w - d_w)]\delta\theta - \delta\omega\sigma_w(1 - \sigma_w)\partial_p\delta\theta.$$

There is an obvious sign error in Yc12 formula: the $\partial_t\sigma_w$ term should have a plus sign, not a minus (if entrainment and detrainment are zero, then an increase of σ_w yields a cooling; since $\delta\theta$ is negative, this is obtained only with a plus sign).

o. Item 15

Equation (31) is derived in a highly heuristic manner and it is hard to follow... The same result is even difficult to obtain by rigorously applying SCA. An exact kinetic energy budget of the SCA system is given by Eq. (A.11). Its vertical integral (assuming steadiness) over the updraft region from the height $z = z_A$ to z_B leads to

$$\left[\sigma_u(\overline{w}_u\overline{K}_u + \rho\overline{w}_u\overline{(w''^2)}_u)\right]_{z_A}^{z_B} = P_{\mathrm{lift}} + P_{\mathrm{buoy}} + \epsilon_u. \tag{2.10}$$

There is some misunderstanding there. What Yc12 calls P_{lift} is an integral over the vertical of a sum of terms representing some effects of entrainment and of pressure anomalies, while what we call P_{lift} in GL10 is the power carried by the updraft at level z_A (i.e., $(1/2)\rho\langle w^3\rangle_A$). We believe that the term P_{lift} of Yc12 is close to zero when GL10 approximations are applied (this is certain for the pressure anomaly terms, since we assume $p' = 0$, and for the $\mathbf{V} - \mathbf{V}_\Gamma$ terms, since we neglect entrainment; it is less clear for the term representing the variation of the cross section of the updraft with altitude).

If we drop the P_{lift} term of Yc12 and bring back $P_{\mathrm{lift}} = (1/2)\rho\langle w^3\rangle_A$ as in GL10, the above equation becomes

$$\left[\sigma_u(\overline{w}_u\overline{K}_u + \rho\overline{w}_u\overline{(w''^2)}_u)\right]\Big|_{(z_B)} = P_{\mathrm{lift}} + P_{\mathrm{buoy}} + \epsilon_u,$$

which is pretty close to our Eq. (31), once various approximations listed in Yc12 are applied.

However, there is still a problem, which is that the $\overline{(w''^2)}_u$ term has a coefficient unity in this equation while it has a coefficient 3/2 in GL10. We believe that this comes from an error made by Yc12 when deriving his kinetic energy equation in section e of the appendix. The error lies in the first step of the kinetic energy equation derivation, namely the multiplication of Eq. (A.9) by $\overline{w}_j$, a step in which a part of the turbulent terms is lost. The right derivation would start by multiplying Eq. (A.8) by w and then proceed with the reduction leading to the evolution equation. Performing the multiplication and averaging in that order makes it possible to use the equality $\langle w^3\rangle = \langle w\rangle^3 + 3\langle w\rangle\langle w''^2\rangle$, while the order chosen by Yc12 corresponds rather to $\langle w\rangle\langle w^2\rangle = \langle w\rangle^3 + \langle w\rangle\langle w''^2\rangle$.

This error can certainly be easily corrected by the authors of Yano et al. (2010). Still it seems to us that SCA would benefit from a clear statement that the segmentally constant approximation applies to all variables except w; hopefully this should not change the equations greatly.

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