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Fatigue of 316L stainless steel notched μm-size components

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The aim of the present paper is to provide an in-depth analysis of the fatigue-life assessment for μm-size 316L stainless steel components. Such components find typical applications in the biomedical field, e.g., in cardiovascular stents. To this purpose, the present work analyzes experimental data on 316L stainless steel from literature for smooth and notched μm-size components using a global computational approach. Several aspects are discussed: (i) the choice of an appropriate constitutive law for cyclic material behavior, (ii) fatigue criteria based on shakedown concepts for finite and infinite lifetime, in particular distinguishing between low, high and very high-cycle fatigue regimes (denoted as LCF, HCF and VHCF, respectively), and (iii) gradient effects in relation with hot-spot as well as average or mean volume approaches for the lifetime estimation. The results give a new insight into the lifetime design of μm-size components and could be directly applied for the fatigue-life assessment of small size structures as, for instance, cardiovascular 316L stainless steel stents.

1. Introduction

In the past fifty years, austenitic stainless steels have been widely and extensively used in several fields, ranging from nuclear or aerospace industries to chemistry or food and beverage processing [1]. In particular, type 316L stainless steel has been largely appreciated for its high ductility and strength under complex thermomechanical loadings, i.e., such a material can reach considerable plastic strains of 0.5–1% at millions of cycles. However, materials like the 316L or 304L stainless steels present a complex material behavior characterized by prismatic and secondary hardening [2–5], which make the design of structures a difficult task. Nowadays, in spite of a large database of models and experimental data [6], both experimental [7–9] and modeling campaigns [10–14] are still continuing in order to improve both safety and performance of the designed structures.

In the recent decades, type 316L stainless steel has gained a privileged position among the materials employed in biomedical devices, e.g., stents, vena cava filters, guide-wires for catheters and pacemaker leads [15,16]. The attractive properties of such a material, e.g., well adapted mechanical characteristics (great ductility, high tensile strength, and a raised elastic limit), biocompatibility, resistance to corrosion as well as fatigue performances, assure the long-term service required by biomedical devices. However, the biomedical field is imposing two specific constraints in the design of 316L stainless steel devices: (i) the interest in the VHCF domain (i.e., $10^7–10^8$ cycles) and (ii) the use of mm- and μm-size components.

As an example, cardiovascular 316L stainless steel stents have to withstand both their initial deployment within the artery, which involves large amounts of plastic deformation, and the long-term service loading induced by the pulsing blood pressure [17]. In particular, such devices must withstand at least 10 years service without failure [18]. Such a constraint translates into 400 million (i.e., $4 \times 10^8$) cycles during stent lifetime, since 70 artery pulses per minute impose 40 million cycles per year.

Moreover, stents are manufactured either through welding of microscopic wires or through laser cutting from thin-walled tubes, both leading to a final truss-type structure composed of struts connected by hinges, with thicknesses in the range of 50–150 μm [19–22]. Stents manufactured through both processes will finally present small radius and thus will be subjected to stress concentrations [23]. Also, μm-size components like stents have length scale in the order of magnitude of the length of grains characterizing their microstructure, i.e., the ratio between the length scales of the mesoscopic structure of grains and of the macroscopic structure of the device, characterized by approximately 10–20 grains across the section, makes the application of standard bulk models inadequate [19–22,24].

Additionally, in spite of the importance of such applications, insufficient (only few) data sets are available in the literature...
2.316L steel: material and fatigue data

The present work focuses on 316L austenitic stainless steel with chemical composition and thermo-mechanical properties given in Tables 1 and 2, see [48] for details.

Before entering into the details of the global computational approach adopted in the present work, this Section describes the...
main results of the two extensive experimental campaigns conducted on 316L specimens of micrometric size by Wiersma et al. [23,25] and Donnelly [26].

2.1. Experimental tests by Wiersma et al.

Wiersma et al. [23,25] carried out a complete experimental campaign on 316L stainless steel specimens taking the form of smooth and notched round/rectangular bars (see Fig. 1(a)-(d) and Tables 3 and 4). Specimens were produced with the standard procedure applied in the stent manufacturing industry: (i) laser cutting; (ii) vacuum anneal treatment or not; and (iii) electro-polishing. The size of the tested specimens was approximately 0.1 mm, thus comparable to that characterizing industrially produced stents. Moreover, the cross-section of the tested specimens contained around 10 grains.

Monotonic tensile test results are represented in terms of stress–strain curves in Fig. 2(a) and mechanical properties are reported in Table 5. The grain size and hardness values provided in Table 5 are identical for the macroscopic and microscopic specimens, confirming that the material itself is identical. The stress–strain curves show that µm-size annealed specimens, when compared to macroscopic specimens, display lower values of yield and ultimate tensile strengths and strain to failure, as well as larger

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Chemical composition (wt%) of 316L austenitic stainless steel [48].</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element</td>
<td>C</td>
</tr>
<tr>
<td>Mass content (%)</td>
<td>0.024</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Thermo-mechanical properties of 316L austenitic stainless steel [48].</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ °C</td>
<td>20</td>
</tr>
<tr>
<td>$E$ $10^3$ MPa</td>
<td>197</td>
</tr>
<tr>
<td>$\sigma$ $10^6$/°C</td>
<td>15.54</td>
</tr>
<tr>
<td>$\tan \delta$ $10^{-6}$/m²·s⁻¹</td>
<td>3.89</td>
</tr>
<tr>
<td>$\lambda$ W/m·°C</td>
<td>14.0</td>
</tr>
</tbody>
</table>

Fig. 1. Geometries of the 316L µm-size notched specimens tested by Wiersma et al. [23,25] (b is the net section; s the thickness; d the notch depth; R the fib-slot; H the hole radius; L the slot length).
Table 3
Summary of the tests on macroscopic 316L specimens and of the fatigue limits by Wiersma et al. [23,25].

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Features</th>
<th>Fatigue ratio (R)</th>
<th>Experimental fatigue limit (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MACRO</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smooth</td>
<td>Round bar ($D = 7.5$ mm, gauge length 25 mm)</td>
<td>0.1</td>
<td>420</td>
</tr>
<tr>
<td>Notched</td>
<td>Round bar ($D = 12$ mm, circumf. V-shaped notch of depth $1$ mm, root radius $0.08$ mm)</td>
<td>0.1</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.8</td>
<td>75</td>
</tr>
</tbody>
</table>

Table 4
Summary of the tests on μm-size 316L specimens and of the fatigue limits by Wiersma et al. [23,25].

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Features</th>
<th>Fatigue ratio (R)</th>
<th>Experimental fatigue limit (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MICRO</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smooth</td>
<td>Rectangular bar (width $0.1$ mm, thickness $0.09$ mm)</td>
<td>0.1</td>
<td>420</td>
</tr>
<tr>
<td>50-notch</td>
<td>Rectangular bar (net section of $0.11$ mm, thickness $0.09$ mm, notch depth $0.05$ mm)</td>
<td>0.1</td>
<td>360</td>
</tr>
<tr>
<td>100-notch</td>
<td>Rectangular bar (net section of $0.11$ mm, thickness $0.09$ mm, notch depth $0.1$ mm)</td>
<td>0.1</td>
<td>355</td>
</tr>
<tr>
<td>145-notch</td>
<td>Rectangular bar (net section of $0.11$ mm, thickness $0.09$ mm, notch depth $0.145$ mm)</td>
<td>0.1</td>
<td>350</td>
</tr>
<tr>
<td>50-fib</td>
<td>Rectangular bar (net section of $0.11$ mm, thickness $0.09$ mm, notch depth $0.05$ mm, slot of $0.001$ mm)</td>
<td>0.1</td>
<td>320</td>
</tr>
<tr>
<td>60-hole</td>
<td>Thin-walled tube (outer diameter $1.7$ mm, wall thickness $0.09$ mm, hole radius $0.06$ mm)</td>
<td>0.1</td>
<td>250</td>
</tr>
<tr>
<td>400-slot</td>
<td>Thin-walled tube (outer diameter $1.7$ mm, wall thickness $0.09$ mm, slot length $0.4$ mm)</td>
<td>0.1</td>
<td>120</td>
</tr>
<tr>
<td>1500-slot</td>
<td>Thin-walled tube (outer diameter $1.7$ mm, wall thickness $0.09$ mm, slot length $1.5$ mm)</td>
<td>0.1</td>
<td>115</td>
</tr>
</tbody>
</table>

Fig. 2. Results from the experiments by Wiersma et al. [23,25]: (a) Stress vs. strain diagram from monotonic tensile tests. Net stress range vs. number of cycles to failure diagram from fatigue tensile tests on (b) smooth macroscopic and μm-size specimens; (c)–(d) notched μm-size specimens.
values of reduction in area before failure, thus demonstrating a size effect. Therefore, one can conclude that for annealed material decreasing specimen size reduces strength but increases ductility. On the contrary, unannealed specimens show a higher strength and lower ductility.

Fatigue tensile tests were carried out using a sinusoidal loading cycle with $R$ ratios varying from 0.1 to 0.8 and are summarized in Tables 3 and 4. Fig. 2(b)–(d) shows fatigue tensile test results in terms of the applied stress range and the number of cycles to failure. The reported applied stress level for specimens with stress concentrations (notches or slots) is a mean stress level computed as the ratio between the applied force and the net area. Tables 3 and 4 summarize the obtained fatigue limits, defined as the values of the stress range for which the average number of cycles to failure is $2 \times 10^6$.

### 2.2. Experimental tests by Donnelly

Donnelly [26] carried out a complete experimental campaign on 316L stainless steel specimens taking the form of smooth rectangular bars (see Table 6). Again, specimens were produced with the standard procedure applied in the stent manufacturing industry: (i) laser cutting; (ii) vacuum anneal treatment; (iii) seamless tubing; and (iv) electro-polishing. The size of the tested specimens ranged from 0.05 to 0.15 mm, thus comparable to that characterizing industrially produced stents. The number of grains per cross-section ranged from 5 to 15.

Fatigue tensile tests were carried out using a push–pull traction cycle with $R$ ratio of 0.5 and are summarized in Table 6. Fig. 3(a)–(b) shows the results of the monotonic and fatigue tensile tests, where the fatigue limit is defined as the value of the stress amplitude for which the average number of cycles to failure is $10^5$. The fatigue limit decreases for the 50 μm specimen, which can be associated to a size effect.

### 3. 316L steel: constitutive law and mechanical analysis

In order to critically investigate the described experimental campaigns, this Section presents the first step of the adopted global computational approach, that is, a mechanical analysis to calculate the shakedown or stabilized mechanical state. In particular, we now describe first our modeling assumptions and the adopted constitutive law; then, we present the mechanical analysis performed with the finite element (FE) method. Models and analyses are realized using the object-oriented FE program, Cast3M (see, e.g., [49]).

#### 3.1. Modeling assumptions

The cyclic behavior of 316L stainless steel shows exceptional ductility and a complex evolution which can be described by using a series of superposed hardening laws (see, e.g., [2–5]).

In the following, we adopt a simplifying view as proposed initially by Skelton [50,51] and successfully applied to LCF lifetime predictions in [10–12,33]. Such a view describes the cyclic material behavior, i.e., the recorded stress level under applied strain amplitude, and identifies three material behavior phases defined by the saturation, tangent, and final points, as shown in Fig. 4: (i) the hardening/softening phase [$0-N_{sat}$] corresponding to the stabilization of the plastic material properties; (ii) the stabilized phase [$N_{sat}-N_{tan}$] represented by a flat plateau with small property variations; and (iii) a final failure phase [$N_{fin}$] characterized by a fast drop of the measured stress (as we impose the displacement/strains). Conventionally, $N_{fin}$ is defined by a 5% or 10% load drop. During the saturation phase we assume that the structure has reached an asymptotic limit cyclic (plastic shakedown) which is described by the constitutive law and can be computed numerically on the structure under scrutiny.

Moreover, we reasonably admit that the mechanical response at $N_f = N_{fin}/2$ lies within the stabilized period and corresponds to material behavior identified for our computational needs (see Fig. 4). This is compatible with the fatigue analysis where characteristics used in the interpretation are equally taken after $N_{fin}/2$ cycles.

Such an approach neglects the finite description of real material behavior (see, e.g., [2–5]) and leads to an approximate error of about 5% in the computed stress level, which is of the order of magnitude of other uncertainties in the modeling. Similar assumptions have already been successfully used for thermo-mechanical fatigue predictions of 304L steel behavior [10–12] or other steels [50,51].

Recalling the discussion of Section 1, another modeling assumption concerns size effects and the application of a continuous model to a structure with sizes close to the grain size, along the line of the methodologies proposed in, e.g., [27,28,31]. We assume

### Table 5
Material properties from monotonic tensile tests by Wiersma et al. [23].

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Grain size (mm)</th>
<th>Yield strength (0.2% proof) (MPa)</th>
<th>Ultimate tensile strength (MPa)</th>
<th>Hardness (Vickers)</th>
<th>Strain to failure</th>
<th>Reduction in area (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MACRO</td>
<td>0.0115</td>
<td>396</td>
<td>754</td>
<td>186</td>
<td>0.60</td>
<td>73</td>
</tr>
<tr>
<td>MICRO</td>
<td>0.0105</td>
<td>315</td>
<td>580</td>
<td>170</td>
<td>0.37</td>
<td>87</td>
</tr>
<tr>
<td>MICRO</td>
<td>(unannealed)</td>
<td></td>
<td></td>
<td>312</td>
<td>0.05</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table 6
Summary of the tests on μm-size 316L specimens and of the fatigue limits by Donnelly [26].

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Features</th>
<th>Fatigue ratio ($R$)</th>
<th>Experimental fatigue limit (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MICRO</td>
<td>Rectangular bar</td>
<td>0.5</td>
<td>120</td>
</tr>
<tr>
<td>50 μm</td>
<td>(width 0.05 mm, thickness 0.06 mm)</td>
<td>0.5</td>
<td>150</td>
</tr>
<tr>
<td>Smooth</td>
<td>Rectangular bar</td>
<td>0.5</td>
<td>150</td>
</tr>
<tr>
<td>75 μm</td>
<td>(width 0.075 mm, thickness 0.06 mm)</td>
<td>0.5</td>
<td>150</td>
</tr>
<tr>
<td>Smooth</td>
<td>Rectangular bar</td>
<td>0.5</td>
<td>150</td>
</tr>
<tr>
<td>100 μm</td>
<td>(width 0.1 mm, thickness 0.06 mm)</td>
<td>0.5</td>
<td>150</td>
</tr>
<tr>
<td>Smooth</td>
<td>Rectangular bar</td>
<td>0.5</td>
<td>150</td>
</tr>
<tr>
<td>150 μm</td>
<td>(width 0.15 mm, thickness 0.06 mm)</td>
<td>0.5</td>
<td>–</td>
</tr>
</tbody>
</table>
that the present analysis will provide a typical 'mean' answer of the structure and that the inherent variability of the properties of the microstructure will provide the spread of the fatigue lifetime. However, such an aspect is not treated here and will be the object of future work.

Moreover, since an important feature of cardiovascular stent applications is given by their small size, where the size of the structure is close to that of the representative volume element, one could reasonably suggest that an appropriate model should include crystal plasticity, as treated in detail by [46,47,30]. However, this could further imply that the microstructure is precisely known, which reduces yield stress upon reversal of loading.

3.2. Adopted constitutive model for 316L stainless steel

The elasto-plastic constitutive model has a J2 yield function and non-linear isotropic and kinematic hardening (with one kinematic center) [6,53–55]. On the one side, isotropic hardening controls the rate at which the stabilized response is achieved, by allowing yield surface expansion and thus, by defining the evolution of stress magnitudes reached in a cyclic stress–strain response; on the other side, kinematic hardening influences the shape of the stabilized hysteresis curve, by allowing yield surface displacement and thus, by introducing a back stress into the cyclic stress–strain response, which reduces yield stress upon reversal of loading.

The multiaxial equation for the back stress tensor, $X$, is defined by:

$$X = \frac{2}{3} C(p) \varepsilon^p - \gamma Xp$$

(1)

with $C(p) = C_0 (1 + (\psi - 1) \exp^{-wp})$ and $p = \sqrt[3]{\frac{2}{3} \varepsilon^p : \varepsilon^p}$, where $\varepsilon^p$ is the plastic strain tensor, $p$ is the accumulated plastic strain, and $C_0, \psi, \gamma$, and $\omega$ are kinematic constants. The isotropic hardening function, $R = R(p)$, is given by the following relation:

$$R = b (R_o - R)p$$

(2)

where $R_o$ is the saturated value of isotropic hardening and $b$ the rate of decay for isotropic hardening. The yield surface, $f = f(\sigma, X, p)$, is defined by the classical function:

$$f = \sqrt{\frac{3}{2} (s - X) : (s - X) - R_0 - R}$$

(3)

where $s$ is the deviatoric tensor for stress and $R_0$ the initial yield stress. The evolution law for the plastic strain, $\varepsilon^p$, takes the form:

$$\varepsilon^p = \frac{3}{2} \frac{s - X}{\sqrt{\frac{3}{2} (s - X) : (s - X) + R}}$$

(4)

In the absence of direct cyclic measurements on the fatigue experiments [25,23,26], we identify model parameters on the experiments by Lê [48], consisting of uniaxial strain-controlled tests with variable amplitude loading on cylindrical specimens. In particular, five variable loading amplitudes have been applied in the following sequence: ±0.30%, ±0.75%, ±2.00%, ±0.75%, and ±0.30%. Table 7 presents the sequence of the applied variable loading amplitudes and the related number of loading cycles; see [48] for further details. Table 8 lists the adopted constitutive model parameters. Fig. 5 shows a comparison between the stabilized

<table>
<thead>
<tr>
<th>Loading amplitude (%)</th>
<th>Number of loading cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>±0.30</td>
<td>25</td>
</tr>
<tr>
<td>±0.75</td>
<td>25</td>
</tr>
<tr>
<td>±2.00</td>
<td>10</td>
</tr>
<tr>
<td>±0.75</td>
<td>40</td>
</tr>
<tr>
<td>±0.30</td>
<td>40</td>
</tr>
</tbody>
</table>
The model successfully captures the experimental loops. The order of magnitude of the numerical stresses and strains with respect to the experimental ones is illustrated in Fig. 6(a)–(b) through a comparison with the monotonic experimental curves obtained by Wiersma et al. [25,23] and Donnelly [26]. As it can be observed, the numerical monotonic and stabilized curves lie within the experimental stress range. However, large errors in terms of stresses (up to 30%) are particularly evident through

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Young Modulus</td>
<td>196,000 MPa</td>
</tr>
<tr>
<td>v</td>
<td>Poisson ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Initial yield stress</td>
<td>211 MPa</td>
</tr>
<tr>
<td>$R_{\infty}$</td>
<td>Saturated value of isotropic hardening</td>
<td>231 MPa</td>
</tr>
<tr>
<td>b</td>
<td>Decay rate for isotropic hardening</td>
<td>6.69</td>
</tr>
<tr>
<td>C</td>
<td>Kinematic constant</td>
<td>67,050 MPa</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Kinematic constant</td>
<td>218</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Kinematic constant</td>
<td>0.73</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Kinematic constant</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Fig. 5. Comparison between the stabilized hysteresis loops generated by the adopted model and the experimental data by Lê [48] for the five applied strain amplitudes listed in Table 7.
the comparison between monotonic curves. This is attributed to
the fact that we adopt a model calibrated on experimental stabilization curves [48], due to the absence of direct cyclic measurements
on the fatigue experiments by Wiersma et al. [25,23] and Donnelly [26]. However, the calibration of the constitutive models against both cyclic and tensile data is important in applications characterized by high initial plastic deformation, which induces high residual or mean stresses, followed by fatigue, such as for a stent where crimping, crimp recoil, deployment and recoil incur plastic deformation, followed by systolic-diastolic pulsatile loading [30,47]. In spite of the overall agreement between predictions and measurements, the preceding calibration of the constitutive law has been applied with precaution to \( \mu \)-size structures. If the structures under scrutiny are, for example, stents, one should take into account (i) that their shape is close to that of wires and we can expect a modification of the exact parameter values as also the difference between the 50 and 150 \( \mu \)m specimens is about 20\%(see Fig. 3) and (ii) that such structures are subjected to large plastic deformation before the fatigue cycling. The operations of crimping, crimp recoil, deployment and recoil, which incur plastic deformation, then followed by systolic-diastolic pulsatile loading [30,47], illustrate our point.

3.3. Mechanical analysis of experiments by Wiersma et al. and Donnelly

A complete series of elasto-plastic FE analyses is performed on each specimen in accordance to the loading cycle of experiments [26,25,23]. In the case of notched specimens, a preliminary analysis is accomplished in order to assess the accuracy of the solution at the notch tip. The mesh refinement is stopped when the elastic solution converges and, therefore, correctly represents the solution.

We consider the notched specimens of Fig. 1(a)-(d) and we generate all the meshes for the specimens taking the form of rectangular bars (i.e., the 50-notch, 100-notch, 145-notch, and 50-fib specimens) using two-dimensional 6-node quadratic plain stress elements, due to the lack of constraint in the thickness direction of these specimens. Then, we generate the meshes of the specimens taking the form of tubes (i.e., the 60-hole, 400-slot, and 1500-slot specimens) using three-dimensional 20-node quadratic brick elements.

According to the symmetry of the problem, we model only one quarter of each specimen, except for the 50-fib specimen for which we model only one half (see Fig. 7(a)–(d)). The finest zone of the mesh corresponds to the notched zone of the specimen and the elements in this part of the mesh are refined depending on the dimension of the notch radius and of the grain size. In particular, specimens are meshed using 4662 elements for the 50-notch, 6582 for the 100-notch, 9068 for the 145-notch, 75,000 for the 50-fib, 61,624 for the 60-hole, 23,346 for the 400-slot, and 25,473 for the 1500-slot (see Fig. 7(a)–(d)). Compared to the grain size, we assume approximately 5 elements per grain size. Consequently, the smallest element size at the root of the notch is characterized by an element size/notch radius ratio ranging between 0.0013 and 0.04. To estimate possible errors due to large spatial gradients, we test finer meshes and we can report that refining mesh above the present limit causes an important increase of computing time without a noticeable improvement of the results (see Section 3.4 for details).

3.4. Results and discussions

As a result of the conducted elasto-plastic FE analyses, we obtain elastic shakedown for the experiments on smooth specimens by Wiersma et al. [25,23] and Donnelly [26] and plastic shakedown for the experiments on notched specimens by Wiersma et al. [25,23].

In order to avoid possible material failures, we first compare the computed strains to the experimental strains to failure (see Table 5) and to the strains of the monotonic curves of Fig. 6(a)–(b). The computed strains lie in the range 0.01–0.1 for all the specimens and loading conditions, thus avoiding possible premature failures.

Fig. 8 shows the distribution of the dissipated energy, \( W^\theta \), at the stabilized cycle, defined as follows:

\[
W^\theta = \int_{\text{cycle}} \sigma : \varepsilon^p \, dt
\]

for the 50-notch specimen of Fig. 1(a), subjected to a net pressure range of 360 MPa (see Fig. 2(c)). As it can be observed, plasticity is localized in a very small region of about 0.01 mm (grain size) around the notch, where \( W^\theta \) assumes a maximum value of about 1 MPa; the remaining part of the specimen presents an elastic shakedown state and no \( W^\theta \) is manifested. The dissipated energy, \( W^\theta \), is also plotted along the edge AB in Fig. 8. Moreover, Fig. 9 shows how mesh refinement does not allow noticeable improvements of the results in terms of dissipated energy.

The 100-notch and 145-notch specimens present similar results since they have the same net section.
Fig. 10 shows the distribution of the dissipated energy, $W_p$, in the 50-fib specimen of Fig. 1(b), subjected to a net pressure range of 320 MPa (see Fig. 2(c)). In such a case, plastic effects are distributed in a very small zone of only $0.001 \text{ mm} \ (1/10 \text{ grain size})$ around the smaller notch (point A), where $W_p$ assumes a maximum value of 6 MPa, evidencing elevated gradient effects. Plastic effects are also visible around the bigger notch (point B) where, however, $W_p$ assumes a maximum value of about 0.5 MPa, comparable to the one obtained for the 50-notch specimen. The dissipated energy, $W_p$, is also plotted along the edge AB in Fig. 10.

The reported observations for the notch specimens are confirmed by the results of a linear elastic analysis conducted on each notched specimen to evaluate the stress concentration factor, $K_c$, defined as the ratio between the peak stress, $\sigma_{\text{peak}}$, at the root of the notch and the nominal stress, $\sigma_{\text{nom}}$, which would be present if a stress concentration did not occur [56]:
with elements of comparable size. This technique permits to assure a coherence and a control of the error in all the cases (see also applications to welded structures in [57,58]). The remark with the constant mesh size applies also to Fig. 12, where we can note a difference in the area affected by the dissipation, i.e., plasticity, even if the hot-spot (point A) reaches more or less the same maximum.

4. 316L steel: fatigue analysis

This Section presents the second step of the adopted global computational approach, i.e., the fatigue analysis to compute the number of cycles before failure. Such an analysis is based on the investigation of the stabilized cycle obtained within the mechanical analysis described in Section 3.3. In particular, depending on the nature of the shakedown cycle, i.e., elastic or plastic, we use a Dang Van [34] or LCF crack initiation [37,38,10–12] criterion, respectively. All the criteria are calibrated on the described experimental campaigns in order to provide a set of criteria that can be used to numerically predict the fatigue-life of cardiovascular 316L stainless steel stents.

4.1. Elastic shakedown: Dang Van criterion

The classical Dang Van (DV) criterion [34] allows to define the limit of the imposed external load under which the structure will have an infinite life.

To this purpose, Dang Van considered the mesoscopic grain scale and the macroscopic structure scale, associated to the notion of representative volume element (RVE). The RVE has to contain a sufficient number of grains to be representative of average macroscopic mechanical properties, as in the present case. If the initial formulation of the DV criterion is not suitable to regions with inhomogeneous or high gradient stresses [47], its extension to welded structures [57,58] or analyses based on grain polycrystalline plasticity [41–44,30,46,47] permit to envision such an outcome which would be equally an entrance point to the VHCF regime.

The relations between mesoscopic and macroscopic fields can be reached, for example, using a homogenization scheme; see, e.g., [59,60]. In particular, by adopting the Lin–Taylor homogenization law, simple passages between the macroscopic and the mesoscopic scale (see, e.g., [61]) allow to state that the lifetime is infinite if the mesoscopic shear stress, \( \sigma_{\text{sl}}(t) \), and the hydrostatic stress, \( \sigma_{\text{h}}(t) \), satisfy the following inequality in all the points of the structure (subscript \( \mu \) stands for mesoscopic variables):

\[
\max \tau_{\mu}(t) + \sigma_{\text{h}}(t) \leq \beta_{\infty}
\]

In particular, we obtain a value of approximately 2.4 for the 50-notch, 100-notch, and 145-notch specimens (in accordance with the values provided in [56]) and of 6 for the 50-fib specimen.

Fig. 11(a)–(b) shows the distribution of the dissipated energy, \( W_p \), in the 60-hole and 1500-slot specimens of Fig. 1(c)–(d), subjected to a net pressure range of 250 and 115 MPa, respectively (see Fig. 2(d)). The values assumed by the dissipated energy are higher for the 1500-slot specimen, notwithstanding the lower load, and additionally, are distributed in a very small zone. In this case, the stress concentration factor, \( K_t \), assumes a value of approximately 3 for the 60-hole, 6 for the 400-slot, and 10 for the 1500-slot specimen.

In Fig. 12, the dissipated energy, \( W_p \), is plotted along the edge AB of the 1500-slot specimen shown in Fig. 11(b), in order to highlight how mesh refinement does not allow noticeable improvements of the results in terms of dissipated energy also for the three-dimensional case. However, in order to assure a coherence between the different computations, we recommend that all the meshes used for both the calibration of the constitutive law or fatigue criteria on the specimens and the structure should be refined in the critical areas

\[
K_t = \frac{\sigma_{\text{peak}}}{\sigma_{\text{nom}}}
\]
where $a_1$ and $b_1$ are material parameters (subscript $\infty$ stands for infinite lifetime). Such parameters are usually deduced from two Wöhler curves related to smooth specimens, giving the fatigue limit in alternated torsion and fully reversed bending. In general, the Wöhler curves are not defined for an infinite number of cycles but rather for $10^6$–$10^7$ cycles. Thus, in the following, we indicate these coefficients as $a_N$ and $b_N$, where subscript $N$ stands for finite lifetime [62–64].

Material parameters, $a_N$ and $b_N$, can be also calibrated by considering two uniaxial cyclic loadings with respective ratios $R_1$ and $R_2$ and fatigue limits $f_{R_1}$ and $f_{R_2}$. In the simple case of a cyclic uniaxial bending loading of $R$ ratio, expressed in the classical form $\sigma_m(t) = \sigma_a \sin \omega t + \sigma_m$, we derive the following equation:

$$\frac{\sigma_a}{2} + a_N \frac{\sigma_a}{3} \left(1 + \frac{1 + R}{1 - R}\right) = b_N \quad \forall R$$

which can be generalized for two different loadings of ratios $R_1$ and $R_2$, as follows:

$$\frac{f_{R_1}}{2} + \frac{a_N}{3} \frac{f_{R_1}}{3} \left(1 + \frac{1 + R_i}{1 - R_i}\right) = b_N \quad \text{for} \quad i = 1, 2$$

The following expressions for $a_N$ and $b_N$ are then obtained:

$$\begin{align*}
a_N &= \frac{f_{R_1}}{2} \left(1 + \frac{1 + R_1}{1 - R_1}\right) = R_1 \left(1 - \frac{R_1}{R_2}\right) \\
b_N &= \frac{f_{R_1} f_{R_2}}{R_1 + R_2} \left(1 + \frac{R_1}{R_2}\right)
\end{align*}$$

The DV parameters, $a_N$ and $b_N$, are calibrated by using the experimental data on the smooth rectangular bars of 75, 100, and 150 μm widths, resulting in an elastic shakedown state (see Tables 4–6). The calibrated DV parameters refer to $10^7$ cycles (i.e., $N = 10^7$) or to an equivalent stent implant period of approximately 8 months, instead of the requested $10^8$ cycles [18]. We however remark that $10^7$ cycles have already been considered as an infinite lifetime as, for example, in [65,66], coherently with the definition given in [25,23,26]. Although specimens tested by Wiersma et al. [23,25] and Donnelly [26] refer to a fatigue-life of $2 \times 10^6$ and $10^7$ cycles, respectively, we could set $N = 10^7$ for the specimens used for the calibration, since the slope of the fatigue curves of

<table>
<thead>
<tr>
<th>Specimen width</th>
<th>Fatigue-life $N$</th>
<th>$a_N$</th>
<th>$b_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>75–150 μm</td>
<td>$10^7$</td>
<td>0.4821</td>
<td>171.4286 MPa</td>
</tr>
<tr>
<td>50 μm</td>
<td>$10^7$</td>
<td>0.4821</td>
<td>137.1429 MPa</td>
</tr>
</tbody>
</table>
ties related to the high gradients of the fields close to the notched energy criterion [10–12]. Moreover, in order to tackle the difficulties related to the high gradients of the fields close to the notched energy criterion [10–12], we construct such a DV line in the hydrostatic-mesoscopic stress plane and loading paths from Fig. 13.

Fig. 2(b) is almost horizontal [23,25]. For safety reasons, we assume a reduced fatigue limit of 400 MPa for the specimens tested by Wiersma et al. [23,25] (see Table 4). Table 9 lists the DV parameters, calibrated on experimental data.

Fig. 13 represents the diagram in terms of the mesoscopic shear stress, $\tau_{\text{mm}}$, and the hydrostatic stress, $\sigma_h$, where the calibrated DV line (red smooth line) and the loading paths generated by the simulations of the experimental specimens of 75.100, and 150 $\mu$m widths are represented. The maximum values for the hydrostatic stress are generated by the specimens tested by Donnelly [26] since presenting high values of mean stress, while the highest mesoscopic shear stresses are generated by the specimen tested by Wiersma et al. [23,25], presenting a higher load amplitude. Additionally, Fig. 13 shows the calibrated DV line (red dot line) related to the specimen of 50 $\mu$m width, which presents a lower value of the fatigue-life due to size effects [26]. As additional data for another $R$ ratio is absent in this case, we construct such a DV line assuming the same slope as in the 75–150 $\mu$m width case. Table 9 also lists the DV parameters, calibrated on 50 $\mu$m width specimens.

4.2. Plastic shakedown: fatigue crack initiation criteria

A fatigue crack initiation criterion is classically defined as a local relation (i.e., in each spatial point of the structure, $x \in \Omega$), between the values of the mechanical fields, $(\varepsilon, \dot{\varepsilon}, \sigma)$, computed for the stabilized cycle, and the number of cycles to failure, $N_f$, of the structure:

$$\max_{x \in \Omega} f(\varepsilon, \dot{\varepsilon}, \sigma) = cN_f^b$$

where $f$ represents the fatigue damage parameter that is characteristic of each criterion, while $b$ and $c$ are material parameters to be calibrated on experimental data.

In the present Section, we focus on the following criteria: (i) the Manson–Coffin criterion [35,36]; (ii) the dissipated energy per cycle criterion [37,38]; and (iii) a modified version of the dissipated energy criterion [10–12]. Moreover, in order to tackle the difficulties related to the high gradients of the fields close to the notched regions, we adopt a volumetric approach in the dissipated energy per cycle criterion. In the following, we present, first, the three criteria and then, we compare the obtained results.

4.2.1. Manson–Coffin fatigue criterion

The Manson–Coffin fatigue criterion is defined as follows [35,36]:

$$\Delta \varepsilon_p = cN_f^b$$

where $f = \Delta \varepsilon_p$ is the amplitude of the plastic strain in the uniaxial case. Since the considered experiments are multiaxial, we extend the amplitude, $\Delta \varepsilon_p$, to this case as follows:

$$\Delta \varepsilon_p = \max \left( \frac{2}{\sqrt{3}} (\varepsilon_p(t_1) - \varepsilon_p(t_2)) \right)$$

where $t_1$ and $t_2$ represent different time instants of the stabilized cycle. The present extension measures the diameter of the plastic strain path.

In the following, we denote the fatigue criterion as $\Phi_{MC}$.  

4.2.2. Dissipated energy per cycle criterion

The criterion based on the dissipated energy per cycle is defined as follows [37,38]:

$$W^p = cN_f^b$$

where $f = W^p$ is the dissipated energy per cycle, integrated in each point of the structure over the complete stabilized cycle, as follows:

$$W^p = \int_{\text{cycle}} \sigma : \dot{\varepsilon}^p \, dt$$

In the following, we denote the fatigue criterion as $\Phi_{Wp}$.

4.2.3. Modified dissipated energy per cycle criterion

The definition of a fatigue parameter depending only on the dissipated energy implies that such a quantity is the only driving force of the damage, even if the effect of mean stress on fatigue is a well known phenomenon. Thus, we adopt the modified dissipated energy criterion proposed by Amiable et al. [10–12], as follows:

$$(W^p + a\sigma_{\text{max}}^\text{sh}) = cN_f^b$$

where $f = (W^p + a\sigma_{\text{max}}^\text{sh})$ is the sum of the dissipated energy, $W^p$, and the maximal hydrostatic stress attended during the stabilized cycle, $\sigma_{\text{max}}^\text{sh} = \max \sigma_h(t)$, with $\sigma_h = 1/3tr(\sigma)$, while $a$ is an additional material parameter.

In the following, the fatigue criterion is denoted as $\Phi_{W_p}$.

4.2.4. Results and discussion

The identification of material parameters, $a$, $b$ and $c$, is a regression problem which can be easily solved using existing solvers (e.g., the fitting tools, cfitool and sfitool, in Matlab).

Such parameters are identified by postprocessing the values of the fatigue damage parameters, $\Delta \varepsilon_p$, $W^p$, and $(W^p + a\sigma_{\text{max}}^\text{sh})$, computed at the root of the notch (hot-spot approach) or around the notch (volumetric approach).

![Fig. 13](Image)

Table 10

Calibrated material parameters, $a$, $b$, and $c$, and obtained correlation coefficient, $R^2$.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Approach</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_{MC}$</td>
<td>Hot-spot</td>
<td>–</td>
<td>–0.4214</td>
<td>0.6199</td>
<td>0.75</td>
</tr>
<tr>
<td>$\Phi_{W_p}$</td>
<td>Hot-spot</td>
<td>–</td>
<td>–0.4687</td>
<td>431.4 MPa</td>
<td>0.74</td>
</tr>
<tr>
<td>$\Phi_{W_p}$</td>
<td>Volume element – 0.005 mm</td>
<td>–</td>
<td>–0.5457</td>
<td>622.5 MPa</td>
<td>0.81</td>
</tr>
<tr>
<td>$\Phi_{W_p}$</td>
<td>Volume element – 0.01 mm</td>
<td>–</td>
<td>–0.6187</td>
<td>907.8 MPa</td>
<td>0.86</td>
</tr>
<tr>
<td>$\Phi_{W_p}$</td>
<td>Volume element – 0.02 mm</td>
<td>–</td>
<td>–0.6124</td>
<td>261.4 MPa</td>
<td>0.68</td>
</tr>
<tr>
<td>$\Phi_{W_p}$</td>
<td>Hot-spot</td>
<td>0.00203</td>
<td>–0.3641</td>
<td>137.1 MPa</td>
<td>0.76</td>
</tr>
</tbody>
</table>
The volumetric approach considers a process volume for fatigue mechanisms \cite{67,68} and assumes such a volume to be a cylinder for the three-dimensional specimens (slot and hole specimens) and a circle for the two-dimensional specimens (notch and fib specimens) with the center corresponding to the root of the notch. In three-dimensional structures, the cylinder is coherent with the spatial distribution of the mechanical fields. Moreover, its height is equal to the thickness of the slot and hole specimens, i.e., \(90 \mu m\). The optimal radius for both the cylinder and the circle is estimated as the minimizer of the statistical correlation coefficient, \(R^2\), measuring the differences between the experimental and the predicted numbers of cycles to failure, as explained in the following.

The approach has been applied to the dissipated energy criterion, \(\Phi_{\text{D}}\). Averaging the dissipated energy inside a process volume is similar to the estimation of the distribution of plastic strain.
localized around the notch. The explored volumes have radii varying from 0.005 to 0.02 mm and the minimizer shows up to be 0.01 mm = 10 μm, which is actually the grain size of the material. Table 10 lists the calibrated parameters, $a$, $b$ and $c$, for the considered criteria. The values related to the volumetric approach are obtained for volume-averaging radii of 0.005, 0.01, and 0.02 mm. Figs. 14(a)–(c)–(d) and 15(a)–(c)–(e) present the diagram in terms of the fatigue parameter and the experimental number of cycles to failure, while Figs. 14(b)–(d)–(f) and 15(b)–(d)–(f) present the comparison between the experimental and the estimated number of cycles to failure for both the hot-spot and volumetric approaches. The results can be placed in a scattered band with a factor of ±3 on the number of cycles to failure, corresponding to the standard deviation.

As a first observation, the lifetimes predicted with the hot-spot approach are not well predicted by all the criteria, while the
volumetric approach related to the 0.01 mm radius gives very good results as the points are inside the ±3 factor on the lifetimes. The experimental number of cycles is often over-predicted with the hot-spot criteria. On the contrary, the hot-spot criteria predict safely lower numbers of cycles to failure for the 50-fib and 1500-slot specimens. As a consequence, one should consider the results obtained by the volumetric approach as a first estimation and then extend quantitatively the analysis in perspective, using one of the techniques proposed in [30,42–44] which explore the stress gradient effect and its interaction with crystal plasticity in a fatigue setting.

As it can be observed from Fig. 14(b)–(d)–(f), the hot-spot approach falls into predicting the fatigue limits of the 50-fib and 1500-slot specimens due to their high stress gradients and plastic strain localization at a distance of only few microns near the notch (see Section 3).

This aspect has been also treated by Wiersma et al. [25,23] who observed that the three notched bar specimens (i.e., the 50-notch, 100-notch, and 145-notch bars) have the same fatigue behavior in terms of net stress range (a fact which might be anticipated from basic notch theory, since the stress field of these notches is almost identical as well), as reported in Table 4. On the contrary, the fatigue limit of the 50-fib specimen is slightly lower. This has important consequences for the design of microscopic components, because it implies that very small root radii can be used in components such as stents, without compromising their fatigue behavior [23]. Slotted tube specimens (i.e., the 60-hole, 400-slot, and 1500-slot tubes) show a decreasing fatigue limit with increasing slot length (see Table 4).

Moreover, we observe that the dissipated energy criterion and its modified version present almost the same results. This can be due to the fact that both these criteria based on values computed at the root of the notch fail in case of high stress gradients. To this purpose, Fig. 16 presents the diagram in terms of the dissipated energy, $W^D$, and the maximal hydrostatic stress, $\sigma_{h}^{\text{max}}$, for all the specimens and shows very high values of the hydrostatic terms for the 50-fib and 1500-slot specimens. We observe also that there is a linear correlation between the dissipated energy and the maximal hydrostatic stress.

To compare accurately the prediction capabilities of the different criteria, we propose to use the correlation coefficient, $R^2$, corresponding to the linear association between experimental and computed fatigue lifetimes. $R^2$ is defined as in standard statistical textbooks [69], as follows:

$$R^2 = 1 - \frac{\sum_{j}(N_j^{\text{exp}} - N_j^{\text{comp}})^2}{\sum_{j}(N_j^{\text{exp}} - \bar{N})^2}$$

(17)

where $N_j$ is the number of cycles related to experiment $j$ (superscripts exp and comp stand for the experimental and the computed values of the number of cycles, respectively), and $\bar{N}$ is the mean of the experimental data.

Table 10 accounts the computed correlation coefficients for each criterion. The obtained values stretch in a reasonable range of reliability for the criteria (see, e.g., [70] for comparison). However, the results related to the hot-spot approach show relative low values of $R^2 = 0.74–0.76$, while the volumetric approach $R^2 = 0.86$ for the volume-averaging radius of 0.01 mm. As it can be observed, the radius of 0.01 mm gives the best correlation coefficient, compared to the 0.005 and 0.02 mm radii. Consequently, all the hot-spot approach criteria are relatively equivalent for the lifetime prediction, while the volumetric approach demonstrates its capability in lifetime prediction of notched µm-size specimens.

Although the selection of the averaging dimension is typically empirical, the proposed volumetric approach allows to give reasonable results in the prediction of crack initiation based on the stress and strain state of a volume of material.

5. Conclusions

The present paper has explored the fatigue of stainless steel components of µm-size by analyzing experimental data from the literature. The specimens were either smooth or notched and presented features similar to those characterizing cardiovascular stents. Our aim was to gain some insight in applications of 316L stainless steel for biomedical devices. These structures of very small sizes (of the order of grain size) are subjected to different environment conditions than standard applications as nuclear piping components. The present paper has investigated several fatigue criteria for infinite and finite lifetime, i.e., the fatigue criteria by Dang Van and Manson–Coffin as well as the dissipated energy per cycle criterion and its modified version, associated with hot-spot as well as volume element approaches. The results have demonstrated that the volume element approach is efficient in the presence of high stress gradients and size effects. The provided criteria, calibrated on suitable experimental data, can be used in the numerical fatigue-life assessment of cardiovascular 316L stainless steel stents. We remark, however, that the proposed approach ignores the inhomogeneous nature of the microstructure.

In order to effectively demonstrate the reliability of the calibrated fatigue criteria and their application to stent design, an ongoing work will propose a computational approach for the fatigue-life assessment of balloon-expandable stents, which combines advanced numerical methods with lifetime prediction methodologies and available material fatigue data. The main idea is to obtain a reliable, general and coherent fatigue prediction methodology through the application of a global computational approach composed of a mechanical finite element analysis, followed by a fatigue analysis. The fatigue analysis will be conducted by applying the proposed criteria depending on the obtained stabilized cycle of the investigated stent.

Moreover, the idea of introducing the material variability as a probability density function will be the object of future work.

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