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Abstract

Cities located in regions prone to natural hazards such as flooding are not uniformly exposed to risks because of sub-city local characteristics (e.g. topography). Spatial heterogeneity thus raises the issue of how these cities have spread and should continue to develop. The current paper investigates these questions by using an urban model in which each location is characterized by a transport cost to the city center and a risk exposure. Riskier areas are developed nearer to the city center than further away. Investment in building resilience leads to more compact cities. At a given distance to the city center, riskier areas have lower land prices and get lower household density and higher building resilience. Actuarially fair insurance generates optimal density and resilience. An increase of insurance subsidization leads to an increase of density in the riskiest areas and a general decrease of resilience. In this case density restrictions and building codes have to be enforced to limit risk over-exposure.

Keywords: urbanization, natural disaster risks, insurance, resilience.

JEL classification: R52, Q54, O18, G22, H23.

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1 Introduction

In October 2012, hurricane Sandy hit the East Coast of the USA, killing 54 people and generating more than 50 billion dollars of losses.[1] The damage was tremendous in Greater New York: 17% of the city was flooded and 150,000 homes were damaged (The Economist, 2012, 2013). Insurance indemnities were paid to affected households that were insured, and relief had to be organized for those that were not covered. People whose houses were destroyed wondered if they should abandon or rebuild them, and if so, how high they should elevate their new homes. Governments wondered if they should authorize development in risky areas like Oakwood Beach on Staten Island, and if so, according to which building codes. Sandy is only one example of extreme meteorological events that have caused large flooding damages in the world in the recent years. Among those, Xynthia superstorm affected the European coast in February 2010, hurricane Katrina struck the New Orleans region in the USA in August 2005 and Maharashtra heavy rains flooded the area of Mumbai in India in July 2005. Each time, these events and their devastating losses have raised the same questions about the necessity of better managing urban development in areas prone to natural hazards.

Most risk-prone regions were initially urbanized because of the many advantages they offered to communities. In particular, many cities are located near seas and/or rivers, as they can provide natural resources and transport facilities. Nowadays, many industries and services rely on these specificities, and agglomeration forces continue to drive urbanization at these locations (Fujita & Thisse, 2002). However, these locations are often double-edged because of exposure to flooding in the case of extreme meteorological events. Natural hazards coupled with expanding urbanization have already increased losses in the last few decades, and these are expected to escalate with the rising sea level and more severe rainfall patterns due to climate change (IPCC, 2014). At a sub-urban scale in risk-prone cities, locations are differentiated not only by their distance to valuable amenities such as the city center but also by exposure to risk due to local characteristics (e.g. topography for flooding risks). The sub-city spatial heterogeneity raises the essential question of how these cities have spread until now and how they should continue to develop in the future.

[1] http://www.emdat.be/

The paper investigates these issues by using an urban model in which each location is characterized by a transport cost to the city center (or to other valuable amenities) and a risk of natural hazard (such as flooding). It focuses on the impacts of risk spatial variation and insurance subsidization on city development.[2] My results are the following. Riskier areas are developed nearer to the city center than further away. Investment in building resilience leads to more compact cities. At a given distance to the city center, riskier areas have lower land prices and get lower household density and higher building resilience. Actuarially fair insurance promotes the optimal development of the city in terms of risk prevention, with optimal household density and optimal building resilience. I analyze how an increase of insurance subsidization affects the city development. If the subsidy is financed by households in the city, it leads to an increase/decrease of density in the riskiest/safest areas. If the subsidy is financed by households in the country, it leads to a general increase of density in the city because it attracts households from other cities. Moreover, in any case, an increase of insurance subsidization leads to a general decrease of building resilience in the city. These results show that density and zoning restrictions as well as building codes have to be enforced in the city to limit risk over-exposure when insurance is subsidized.[3]

Academics in insurance economics have shown much interest in natural disasters, in particular because of the numerous imperfections in natural disaster insurance markets (Kunreuther, 1984; Kunreuther & Michel-Kerjan, 2009). On the supply side (Charpentier & Le Maux, 2014; Jaffee & Russell, 1997), diversification issues lead private insurers to supply contracts at prices largely above actuarially fair rates. On the demand side (Botzen et al., 2015; Kunreuther et al., 2007; Raschky & Weck-Hannemann, 2007), households under-insure even if insurance is fair, in particular because they under-estimate the risk or they expect free assistance (charity hazard). In

[2]In the present framework, households deliberately purchase full insurance because they are risk-averse and insurance is supplied at or below actuarially fair prices. The model does not consider charity hazard or risk perception bias. Note however that the expectancy of assistance or the under-estimation of risk should have effects similar to insurance subsidization on the city development in terms of risk over-exposure.

[3]Density restriction at one location consists in limiting urban density while zoning restriction at one location consists in completely forbidding urban development. Building codes consist in imposing a minimal level of building resilience.

this context, policy makers have implemented natural disaster public policies such as the National Flood Insurance Program (NFIP) in the USA and various programs in Europe like the CatNat in France (Bouwer et al., 2007; Kunreuther & Michel-Kerjan, 2009). To deal with diversification issues, public insurance/relief can complement the weak private insurance supply (e.g. in the USA) or public reinsurance can help private insurance to supply contracts at lower prices (e.g. in France). However, these policies cannot solve the weak insurance demand issues without subsidizing insurance or/and making it mandatory. For instance, the NFIP in the USA subsidizes contracts in risky areas thanks to taxpayers, and insurance is requested for access to loans. Meanwhile, the CatNat Program in France subsidizes contracts in risky areas with the other contracts and insurance is mandatory to avoid adverse selection. If the advantage of subsidization is to improve insurance demand and risk sharing (Browne & Hoyt, 2000; Grace et al., 2004), the disadvantage is to lead to risk over-exposure because it does not provide the right incentives for individual risk prevention (Bagstad et al., 2007; Courbage et al., 2013; Picard, 2008).

Academics in urban economics have focused on natural disaster issues in the context of city development. As modeled first by Alonso (1964), households spread out in the space surrounding the city center to commute there for consumption or work, and those settled further away incurring higher transport costs are compensated by lower land rent, which explains the increasing housing lot sizes and the decreasing density with distance to the city center. Polinsky & Shavell (1976) and Scawthorn et al. (1982) add in their model the existence of a negative amenity such as exposure to natural hazard. These models show that, at a given distance from the city center, the land price decreases when the loss exposure increases. Many empirical studies have confirmed this effect for natural disaster risks, as summarized in the meta-analysis by Daniel et al. (2009). Because households do not want to incur too much transport cost or natural disaster cost, Frame (1998) demonstrates that riskier areas are developed nearer to the city center than further away, and some risky areas inside the city outer boundary may stay undeveloped. The tradeoff between transport cost and natural disaster cost has been observed empirically for instance by Smith (1993) and Atreya & Czajkowski (2014). Frame (1998) also points out that insurance subsidization decreases the land price difference between risky areas and safe areas, as confirmed empirically by Shilling et al. (1989). Furthermore, Frame (2001) shows theoretically that risk aversion can lead households to under-develop risky areas. However, many empirical studies, such as Browne & Hoyt (2000), Harrison et al. (2001) and Michel-Kerjan et al. (2012), suggest that households are more inclined to risk over-exposure because of insurance subsidization, risk under-estimation or charity hazard, than to risk under-exposure because of risk aversion.[4] In this case, urban regulation should be enforced to limit over-exposure, in particular in terms of zoning/density restrictions and building codes (Bagstad et al., 2007; Kunreuther, 1996; Kunreuther & Michel-Kerjan, 2013). In an urban theoretical model with risk exposure but no transport costs, Grislain-Letrémy & Villeneuve (2014) show that zoning restrictions can be Pareto improving in the case of full insurance subsidization. In empirical analysis, Czajkowski & Simmons (2014) and McKenzie & Levendis (2010) respectively observe that investments in building resilience reduce natural disaster losses and increase housing values.

The present paper aims to further analyze the role of natural hazard exposure and insurance subsidization in the development of risk-prone cities with transport costs. Relative to the previously cited theoretical papers on urban economics, the present paper adds building resilience modeling and analyzes how densities and resiliences are affected by natural hazard exposure and insurance subsidization. This analysis is essential from the perspective of implementing efficient urban regulation, in terms of zoning restrictions, density restrictions and building codes, for cities with transport costs and natural disaster risks. The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 provides an analysis of city development. Section 4 provides an analysis of the impact of a change in insurance subsidization. Section 5 concludes.

2 Risk-prone city model

I consider a static model of a city with commuting transport costs and natural hazard exposure, in the spirit of Frame (1998, 2001), Polinsky & Shavell

[4]Browne & Hoyt (2000) observe that households do not usually buy insurance at fair prices, Harrison et al. (2001) notice that the housing rent difference between risky and safe areas is below the expected loss difference and Michel-Kerjan et al. (2012) point out that insured households let their insurance contract lapse after a few years even when those are below fair prices. All this would not be possible if risk aversion was the dominating factor.

(1976) and Scawthorn et al. (1982). The city is inhabited by N identical households.[5] The sub-city scale grid is modeled by a two-dimensional continuous space with the coordinate system $x = (x_1, x_2)$. Because of spatial heterogeneity due to transport costs and natural hazards, all variables potentially depend on location x. Moreover, each variable has a unique value at each location x because I consider identical households and identical housing developers. The city has a pre-established center located at $x = (0, 0)$, also called the central business district where work and consumption activities are concentrated.

Households compete to spread out in the space around the city center and commute between their housing location and the city center. They choose their housing location x and the quantity of goods purchased in the city center, aggregated in a composite good denoted $z(x)$. Besides composite good consumption, households value their housing good consumption, characterized by lot size, measured in land area unit and denoted $s(x)$.[6] The utility function of each household, denoted $v(.)$, depends on $z(x)$ and $s(x)$ and is classically supposed to be twice continuously differentiable, strictly increasing in each argument (with $\partial_z v(0, s) = \infty$ and $\partial_s v(z, 0) = \infty$) and globally concave. The composite good supplied in the city center is considered as the numéraire (i.e. price equal to 1 for one unit of good) and the housing good supplied by housing developers at location x has a housing unit price denoted $p_h(x)$ (i.e. price for one land area unit with housing). The composite good expenses and the housing rent for one household located at x are thus respectively $z(x)$ and $p_h(x)s(x)$.

Besides composite good expenses and housing rent, households incur commuting transport costs and expenses related to natural hazards. One household settling at location x incurs the given transport cost $t(x)$ because of commuting between its housing location and the city center (or potentially other valuable amenities). For example, a city located next to an estuary is depicted in figure 1.[7] On the land, the darkness of the square

[5]I consider identical households in order to analyze the average development of the city. Inequality or asymmetric information issues are not the purpose of the analysis.

[6]The lot size for one household is the land area for this household. For example, for a building occupying $400m^2$ of land and inhabited by 10 households, the lot size of one household is $40m^2$.

[7]In figures 1 and 2, the space is represented by a discrete grid even if the model is continuous.

units characterizes the commuting transport cost $t(x)$ for each household located at x. Darker areas represent locations further from the city center with higher transport costs. In stylized models, transport costs are often considered to be proportional to the distance to the city center. However, real transport costs are more complex than this stylized form, in particular because of transport system complexity. Moreover, other potential amenities (e.g. the positive amenities of being near the water-front) should be taken into account in the transport costs. Note also that transport costs should include different costs, in particular the direct transport cost but also the time opportunity cost.

One household settling at location x is also exposed to natural hazards (such as flooding), with the given probability of impact $\pi(x)$. The level of the loss in case of impact, denoted $l(.)$, depends on the housing lot size $s(x)$ and on the building resilience, denoted $b(x)$. The loss function $l(.)$ is assumed to be twice continuously differentiable. It is decreasing with b at a decreasing rate because the most efficient resilience investments are made first. Besides, if it is reasonably assumed that more households on a land unit leads to more total losses on this land unit (for a given building resilience level), the loss function is such that $\frac{l(s,b)}{s} \geq \frac{\partial l}{\partial s}(s,b)$ for any s and b.[8] Note that losses should include direct and indirect losses. The city depicted in figure 1 is also represented in figure 2 for natural hazard exposure. On the land, the darkness of the square units characterizes the probability $\pi(x)$ of being affected by a natural hazard for each household located at x. The higher the risk, the darker the location. For flooding risks, locations at lower altitude are usually more subject to flooding and should be darker. The probability of being affected by a natural hazard can correspond for example to the probability that the water level reaches a threshold level that induces significant losses for households. Besides, I consider that insurance is supplied to households at or below fair prices because I do not consider any insurance transaction cost and I consider potential insurance subsidy. As households are risk-averse (i.e. their utility function is concave), they deliberately purchase full insurance coverage and bear a certain cost related

[8]With $\frac{1}{s}$ households on a developed land unit, each one having a lot size s, the total loss on the land unit is $\frac{1}{s}l(s,b)$. If more households on a land unit leads to more total losses on this land unit, the loss function is such that $\frac{l(s_1,b)}{s_1} \leq \frac{l(s_2,b)}{s_2}$ for any $\frac{1}{s_1} \leq \frac{1}{s_2}$ and b. In this case, for any s and $ds \geq 0$, $\frac{l(s+ds,b)}{s+ds} \leq \frac{l(s,b)}{s}$, which leads to $s\big(l(s,b) + \frac{\partial l}{\partial s}(s,b)ds\big) \leq (s+ds)l(s,b)$ and then $\frac{\partial l}{\partial s}(s,b) \leq \frac{l(s,b)}{s}$ with a first order development.

to natural disaster risks, which is the insurance premium. With insurance subsidy corresponding to a fraction $\lambda \in [0,1]$ of expected losses, the premium paid by a household located at x is $(1-\lambda)\pi(x)l(s(x), b(x))$. The higher λ, the higher the subsidy. The insurance subsidy can be financed either by the city through a lump-sum tax on household wealth or by another party outside the city. In the former case, the tax borne by each household in the city is $\overline{\tau} = \frac{\lambda}{N} \iint \pi(x) l\big(s(x), b(x)\big) n(x) dx_1 dx_2$ (in which $n(x)$ is the household density at location x). In the latter case, the tax borne by each household in the city is $\overline{\tau} = 0$.[9] For one household located at x, transport cost, insurance premium and tax are thus respectively $t(x)$, $(1-\lambda)\pi(x)l(s(x), b(x))$ and $\overline{\tau}$.

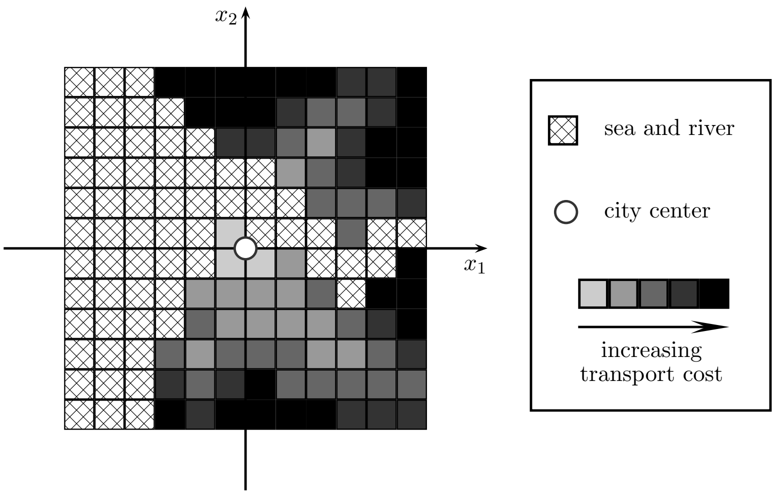

Figure 1: Commuting transport costs in the city.

As households are identical in terms of preferences and wealth, denoted $\overline{y}$, they reach the same utility level $\overline{v}$ at equilibrium. Otherwise, households with lower utility levels at location x would have settled at location x' where other households reach a higher utility level, which would have decreased at x and increased at x' housing unit price until the equilibrium with spatially uniform utility level had been reached. Following Alonso (1964) and Fujita & Thisse (2002), competition between households over where to settle leads

[9]The latter case is representative of an insurance subsidized by the country which is large relative to the city. Besides, note that a natural disaster like flooding usually strikes many locations of a city at the same time and thus has an aggregate risk component at the city level. However, an insurance system organized at the country level enables to better diversify risk.

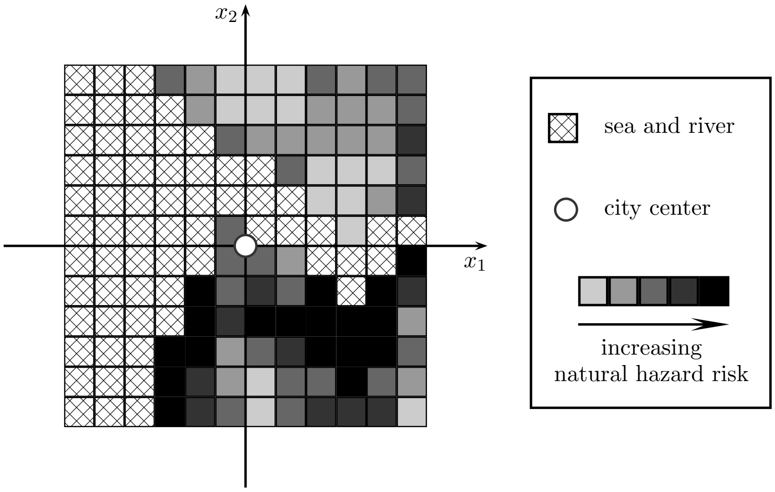

Figure 2: Natural hazard risks in the city.

housing prices to be the solutions of bid price problems: at each location x, the housing unit price $p_h(x)$ corresponds to the highest price that can be afforded by households. The wealth minus expenses except housing rent, divided by the lot size, is the maximal amount that can be paid by one household for one land unit with housing. As households are free to choose their composite good consumption and reach the utility level $\overline{v}$, the housing unit bid price problem at location x can then be expressed as follows:

$$\begin{aligned} p_h(x) = \max_{z(x)} \; & \frac{\overline{y} - \overline{\tau} - z(x) - t(x) - (1-\lambda)\pi(x) l\big(s(x), b(x)\big)}{s(x)} \\ & s.t. \; v\big(z(x), s(x)\big) = \overline{v}. \end{aligned} \tag{1}$$

Housing goods are supplied by identical housing developers in competition. Housing developers observe the housing unit price (1) resulting from the competition between households. They compete to acquire land from absentee land owners at the land unit price denoted $p_l(x)$ at each location x (i.e. price for one land area unit without housing). They choose the housing lot size $s(x)$ and the building resilience $b(x)$ for urban development at each location x. Besides the cost of land, they incur the cost of housing lot development, denoted $c(s(x), b(x))$ for lot size $s(x)$ and resilience $b(x)$ for one household. The cost function $c(.)$ is assumed to be twice continuously differentiable. It is increasing with b at an increasing rate because the less costly resilience investments are made first. Besides, if it is reasonably assumed that more households on a land unit leads to more total housing development costs on this land unit (for a given building resilience level), the cost function is such that $\frac{c(s,b)}{s} \geq \frac{\partial c}{\partial s}(s,b)$ for any s and b.[10] At each location, housing developers are constrained by the availability of one land area unit. Similarly to the housing unit price, the land unit price $p_l(x)$ is determined at each location x by the highest price that can be afforded by housing developers because of competition. The housing rent per household minus the development cost, multiplied by the household density denoted $n(x)$, is the maximal amount that can be paid by one housing developer for one land area unit. As housing developers are free to choose housing lot sizes and building resiliences, and they observe the housing unit price (1) and face land constraints, the land unit bid price problem at location x can then be expressed as follows:

$$\begin{gathered} p_l(x) = \max_{s(x),b(x)} \Big(p_h(x)s(x) - c\big(s(x), b(x)\big)\Big)n(x) \\ s.t.\ (1) \text{ and } n(x)s(x) \leq 1. \end{gathered} \tag{2}$$

The boundaries of the city correspond to the locations where the land unit price $p_l(x)$ is equal to the land opportunity rent denoted $\overline{p_a}$ (e.g agricultural rent).

Finally, the city is characterized by its number of households:

$$N = \iint n(x)dx_1dx_2. \tag{3}$$

With a given number of households (i.e. N given), (3) indirectly determines the welfare level $\overline{v}$ in the city. This characterizes in particular a "closed city" in terms of population. With a given welfare level (i.e. $\overline{v}$ given), (3) determines the number of households N in the city. This characterizes in particular an "open city" in which the welfare level depends on the welfare level outside the city.

3 Risk-prone city development

Outside the boundaries of the city, housing development is not profitable ($n(x) = 0$ and $p_l(x) = \overline{p_a}$). On the boundaries, land may be partly developed ($0 \leq s(x)n(x) \leq 1$) because housing development is equally profitable to

[10] The proof is similar to the one in the footnote 8 for the natural disaster loss function.

agriculture ($p_l(x) = \overline{p_a}$). Inside the boundaries, land is fully developed because housing development is more profitable than agriculture and thus the household density is:

$$n(x) = \frac{1}{s(x)}. \tag{4}$$

As explained in the previous section, households settled in the city reach the same welfare level $\overline{v}$ at equilibrium. As the utility function $v(.)$ is strictly increasing in z, $\tilde{z}(s,v)$ can be defined such that $v(\tilde{z}(s,v),s) = v$ and $\tilde{z}(.)$ is decreasing with s at a decreasing rate because $v(.)$ is concave (proof in appendix A.1). Thus, the composite good consumption $z(x)$ purchased by one household settled at location x can be expressed as a function of the housing lot size $s(x)$ and the uniform welfare level $\overline{v}$:

$$z(x) = \tilde{z}\big(s(x), \overline{v}\big). \tag{5}$$

With (5), the housing unit bid price problem (1) boils down to the housing unit price:

$$p_h(x) = \frac{\overline{y} - \overline{\tau} - \tilde{z}\big(s(x), \overline{v}\big) - t(x) - (1-\lambda)\pi(x) l\big(s(x), b(x)\big)}{s(x)}. \tag{6}$$

With the housing unit price (6) and the household density (4), the land unit bid price problem (2) boils down to:

$$p_l(x) = \max_{s(x), b(x)} \frac{\overline{y} - \overline{\tau} - \tilde{z}\big(s(x), \overline{v}\big) - t(x) - (1-\lambda)\pi(x) l\big(s(x), b(x)\big) - c\big(s(x), b(x)\big)}{s(x)}. \tag{7}$$

The housing lot size $s(x)$ and the building resilience $b(x)$ chosen by housing developers at location x inside the boundaries of the city are the solutions of the first order conditions of (7) (proof in appendix A.2):

$$\frac{\partial_s v}{\partial_z v}\big(s(x), \tilde{z}(s(x), \overline{v})\big) = (1-\lambda)\pi(x)\frac{\partial l}{\partial s}\big(s(x), b(x)\big) + \frac{\partial c}{\partial s}\big(s(x), b(x)\big) + p_l(x), \tag{8}$$

$$-(1-\lambda)\pi(x)\frac{\partial l}{\partial b}\big(s(x), b(x)\big) = \frac{\partial c}{\partial b}\big(s(x), b(x)\big). \tag{9}$$

(8) states that the housing lot size $s(x)$ for one household at location x is chosen such that it equalizes the marginal rate of substitution to the marginal housing unit cost (over the composite good price, i.e. the numéraire). The marginal rate of substitution characterizes the marginal benefit of increasing the housing lot size for the household, which decreases from $+\infty$ to 0 when $s(x)$ increases from 0 to $+\infty$. The marginal housing unit cost is composed

of the marginal insurance premium borne by the household, the marginal housing development cost and the land unit price. (9) relates that the building resilience $b(x)$ for housing at location x is chosen such that it equalizes the marginal benefit of decreasing insurance premium for the household to the marginal cost of increasing building resilience for the housing developer. Note that if $-(1-\lambda)\pi(x)\partial_b l(s(x),0) \leq \partial_b c(s(x),0)$, $b(x)$ is binding in 0. With $s(x)$ and $b(x)$ being determined by (8) and (9), (7) then indirectly gives the land unit price:

$$p_l(x) = \frac{\overline{y} - \overline{\tau} - \tilde{z}\big(s(x), \overline{v}\big) - t(x) - (1-\lambda)\pi(x) l\big(s(x), b(x)\big) - c\big(s(x), b(x)\big)}{s(x)}. \quad (10)$$

Proposition 1 *With null cross derivation for $l(.)$ and $c(.)$ relative to their two arguments, the housing lot size $s(x)$ and the building resilience $b(x)$ vary in space as follows:*[11]

$$A_1(x)\frac{ds}{d\overrightarrow{x}} = \frac{1}{s(x)}\frac{dt}{d\overrightarrow{x}} + (1-\lambda)\left(\frac{l\big(s(x), b(x)\big)}{s(x)} - \frac{\partial l}{\partial s}\big(s(x), b(x)\big)\right)\frac{d\pi}{d\overrightarrow{x}}, \quad (11)$$

$$A_2(x)\frac{db}{d\overrightarrow{x}} = -(1-\lambda)\frac{\partial l}{\partial b}\big(s(x), b(x)\big)\frac{d\pi}{d\overrightarrow{x}}, \quad (12)$$

in which $A_1(x)$ and $A_2(x)$ are positive.

Proposition 1 is proved in appendix A.3. (11) tells that, at a given risk of natural hazard, the housing lot size $s(x)$ increases while translating further away from the city center. Thus, the household density $n(x)$ (i.e. the inverse of the housing lot size $s(x)$) decreases while translating further away from the city center, as first explained by Alonso (1964). With the reasonable assumption $\frac{l(s,b)}{s} \geq \frac{\partial l}{\partial s}(s,b)$ for any s and b, the coefficient in front of $\frac{d\pi}{d\overrightarrow{x}}$ in (11) is positive and (11) says that, at a given distance to the city center, the housing lot size $s(x)$ increases while translating towards riskier areas if insurance is not fully subsidized ($\lambda < 1$). Thus, the household density $n(x)$ decreases while translating towards riskier areas in this case. As $l(.)$ is decreasing with b, (12) points out that the building resilience increases while translating towards riskier areas if insurance is not fully subsidized ($\lambda < 1$).

Proposition 2 *The housing unit price $p_h(x)$ and the land unit price $p_l(x)$ vary in space as follows:*

$$\frac{dp_h}{d\overrightarrow{x}} = \frac{dp_l}{d\overrightarrow{x}} + \frac{\frac{\partial c}{\partial s}(s(x), b(x)) - \frac{c(s(x),b(x))}{s(x)}}{s(x)}\frac{ds}{d\overrightarrow{x}} + \frac{\frac{\partial c}{\partial b}(s(x), b(x))}{s(x)}\frac{db}{d\overrightarrow{x}}, \quad (13)$$

[11] $d\overrightarrow{x}$ corresponds to any small move in space: $d\overrightarrow{x} = (dx_1, dx_2)$.

$$\frac{dp_l}{d\vec{x}} = -\frac{1}{s(x)}\frac{dt}{d\vec{x}} - \frac{(1-\lambda)l(s(x),b(x))}{s(x)}\frac{d\pi}{d\vec{x}}. \tag{14}$$

In proposition 2, (14) is obtained by spatial derivation of (7) with the envelop theorem, while (13) is obtained by spatial derivation of the combination of (6) and (10). (14) relates firstly that, at a given risk of natural hazard, the land unit price $p_l(x)$ decreases while translating further away from the city center, as first explained by Alonso (1964). (14) tells secondly that, at a given distance to the city center, the land unit price $p_l(x)$ decreases while translating towards riskier areas if insurance is not fully subsidized ($\lambda < 1$). Moreover, the higher the insurance subsidization (λ), the lower the land price difference between risky areas and safe areas. These observations confirm the results of Frame (1998), Polinsky & Shavell (1976) and Scawthorn et al. (1982) in a context including building resilience. (13) indicates that the housing unit price $p_h(x)$ is modified through three channels while moving in the city: the land unit price $p_l(x)$, the housing lot size $s(x)$ and the building resilience $b(x)$. At a given risk of natural hazard, the housing unit price $p_h(x)$ decreases while translating further away from the city center because firstly the land unit price decreases, secondly the effect through the housing lot size is negative (because $ds/d\vec{x} \geq 0$ and with the reasonable assumption $\frac{c(s,b)}{s} \geq \frac{\partial c}{\partial s}(s,b)$) and thirdly the effect through the building resilience is null. At a given distance to the city center, translating towards riskier areas leads to the decrease of housing unit price $p_h(x)$ through the decrease of land unit price $p_l(x)$ and the increase of housing lot size $s(x)$ (as far as $\frac{c(s,b)}{s} \geq \frac{\partial c}{\partial s}(s,b)$), while on the other hand it leads to the increase of housing unit price $p_h(x)$ through the increase of building resilience $b(x)$. This differentiates slightly the spatial variation of land unit price $p_l(x)$ and housing unit price $p_h(x)$, contrary to the previously cited papers which do not consider building resilience. Moreover, it is coherent with the empirical observation by McKenzie & Levendis (2010) that investments in building resilience increase housing prices.

Proposition 3 *If the probability of natural hazard is denoted $\pi^*(t)$ on the city boundaries, $\pi^*(t)$ is such that:*

$$\frac{d\pi^*}{dt} = -\frac{1}{(1-\lambda)l(s(x),b(x))}. \tag{15}$$

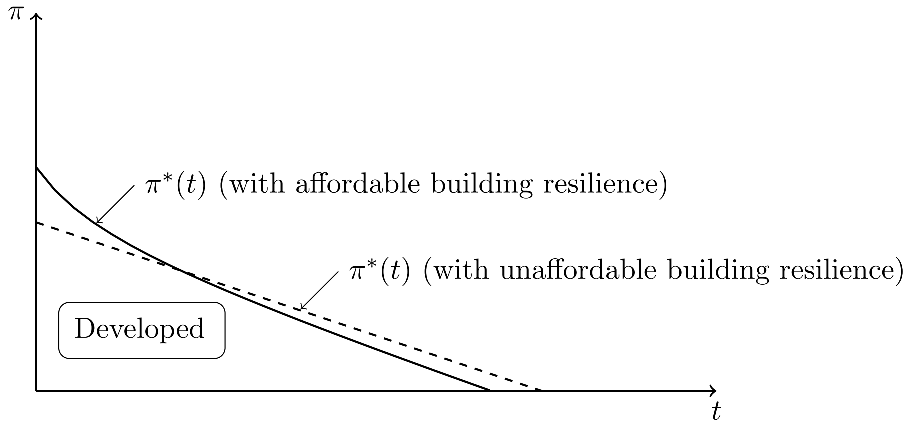

Figure 3: City boundaries and developed areas, as a function of transport cost t and probability π of natural hazard.

Proposition 3 is directly deduced from (14) because the land unit price $p_l(x)$ is constant and equal to the opportunity rent $\overline{p_a}$ on the city boundaries. (15) expresses that riskier locations are developed near the city center because of lower transport cost, which confirms the result of Frame (1998) in a context including building resilience. A location x at a distance t from the city center is developed if $\pi(x) \leq \pi^*(t)$. The outer boundary of the city corresponds to the developed area the furthest away from the city center. The inner boundaries of the city correspond to the riskiest developed area for each distance to the city center. Figure 3 illustrates on a graph, with transport and risk as coordinates, the city boundaries and the developed areas. The slope of π^* relative to t is steeper when building resilience is implemented. Thus, more households are located near the city center (i.e. the city is more compact) when building resilience is more affordable.

4 The impact of insurance subsidization

Proposition 4 *With actuarially fair insurance ($\lambda = 0$), the allocation of resources is Pareto optimal.*

Proposition 4 is proved in appendix B.1. Actuarially fair insurance policy leads to the Pareto optimal allocation of resources because it gives the right incentives to households and housing developers in terms of density development and building resilience. In practice, actuarially fair insurance is hardly ever implemented, and policy makers usually implement insurance subsidy.

Proposition 5 *With null cross derivation for $l(.)$ and $c(.)$ relative to their two arguments, the increase of insurance subsidization has the following impact on urban development at each location x of the city:*

$$s(x)A_1(x)\frac{ds(x)}{d\lambda} = \pi(x)s(x)\left(\frac{\partial l}{\partial s}(s(x),b(x)) - \frac{l(s(x),b(x))}{s(x)}\right) + \alpha(x)\frac{d\overline{v}}{d\lambda} + \frac{d\overline{\tau}}{d\lambda}, \tag{16}$$

$$A_2(x)\frac{db(x)}{d\lambda} = \pi(x)\frac{\partial l}{\partial b}(s(x),b(x)), \tag{17}$$

in which $\alpha(x) = \frac{\partial \tilde{z}}{\partial v}(s(x),\overline{v}) - s(x)\frac{\partial^2 \tilde{z}}{\partial v \partial s}(s(x),\overline{v})$ and $A_1(x)$ and $A_2(x)$ are positive.

Proposition 5 is proved in appendix B.2. (16) characterizes how the increase of insurance subsidy (λ) affects the housing lot size $s(x)$ and thus the household density $n(x)$ at each location in the city. The direct impact corresponds to the first term on the right-hand side of (16), which is negative with the reasonable assumption $\frac{l(s,b)}{s} \geq \frac{\partial l}{\partial s}(s,b)$ for any s and b. A given increase of λ gives, through this direct effect, a density increase which is proportional to the probability $\pi(x)$. The indirect impact through the levels of welfare $\overline{v}$ and tax $\overline{\tau}$ corresponds to the second and third terms. If the number N of households is fixed (which characterizes a "closed city" with $\overline{v}$ endogenously determined), the density cannot increase everywhere in the city[12] and the increase of insurance subsidy reallocates households from safer areas to riskier areas because of the direct effect. If the welfare level $\overline{v}$ is fixed (which characterizes an "open city" with N endogenously determined), $\overline{v}$ is not affected by an increase of λ while the impact on $\overline{\tau}$ depends on who bears the cost of insurance subsidization. If the households in the city bear this cost through the lump-sum tax $\overline{\tau} = \frac{\lambda}{N}\iint \pi(x)l\big(s(x),b(x)\big)n(x)dx_1dx_2$, an increase of $\overline{\tau}$ due to an increase of λ makes the city less attractive, which explains why it increases lot sizes and decreases densities. Thus, in this case, the increase of insurance subsidy leads to a density increase in strongly risky areas and a density decrease in weakly risky areas. If the households in the city do not bear the cost of insurance subsidization

[12]With N fixed, the population constraint (3) gives $0 = \iint \frac{1}{s(x)^2}\frac{ds(x)}{d\lambda}dx_1dx_2$, which means that $\frac{ds(x)}{d\lambda}$ cannot be negative at all locations. Thus, $\alpha(x)\frac{d\overline{v}}{d\lambda} + \frac{d\overline{\tau}}{d\lambda}$ in (16) cannot be negative for all the location in the city.

(i.e. $\overline{\tau} = 0$), an increase of λ does not have this negative effect on the city attractiveness. Thus, in this case, the increase of insurance subsidy leads to a general density increase in the city. These results explain in which direction density policies should be enforced in risk-prone cities when insurance subsidy is implemented. Besides, (17) points out how the increase of insurance subsidy (λ) modifies the building resilience $b(x)$ at each location in the city. The impact is negative and proportional to the local probability $\pi(x)$. A given increase of λ leads to a higher building resilience decrease in risky areas than in safe areas. As a consequence, whether with a closed city or an open city and whoever subsidizes insurance, the increase of insurance subsidy leads to a general decrease in building resilience in the city. Note that this decrease is null if the building resilience is already binding in zero (which is the case at a risk-free location). These results show that resilience policies should be enforced when insurance subsidy is implemented.

Proposition 6 *The increase of insurance subsidization has the following impact on housing and land prices at each location x in the city:*

$$\frac{dp_h(x)}{d\lambda} = \frac{dp_l(x)}{d\lambda} + \frac{\frac{\partial c}{\partial s}(s(x),b(x)) - \frac{c(s(x),b(x))}{s(x)}}{s(x)}\frac{ds(x)}{d\lambda} + \frac{\frac{\partial c}{\partial b}(s(x),b(x))}{s(x)}\frac{db(x)}{d\lambda}, \tag{18}$$

$$\frac{dp_l(x)}{d\lambda} = \frac{1}{s(x)}\left(\pi(x)l(s(x),b(x)) - \beta(x)\frac{d\overline{v}}{d\lambda} - \frac{d\overline{\tau}}{d\lambda}\right), \tag{19}$$

in which $\beta(x) = \frac{\partial \tilde{z}}{\partial v}\big(s(x),\overline{v}\big)$ is positive.

In proposition 6, (19) is obtained at a given location by derivation of (7) relative to λ with the envelop theorem, while (18) is obtained at a given location by derivation of the combination of (6) and (10) relative to λ. (19) relates how the increase of insurance subsidy (λ) affects the land price $p_l(x)$ at each location in the city. The direct impact corresponds to the first term on the right-hand side which is positive and proportional to the local probability $\pi(x)$. For a given increase of λ, the riskier the location, the higher the land price increase through this direct effect. The indirect impact through the levels of welfare $\overline{v}$ and tax $\overline{\tau}$ corresponds to the second and third terms. If the welfare level $\overline{v}$ is fixed (which characterizes an "open city" with N endogenously determined), $\overline{v}$ is not affected by an increase of λ while the impact on $\overline{\tau}$ depends on who bears the cost of insurance subsidization. If the households in the city bear this cost through the lump-sum tax $\overline{\tau} = \frac{\lambda}{N}\iint \pi(x)l\big(s(x),b(x)\big)n(x)dx_1dx_2$, an increase of $\overline{\tau}$ due to an increase of λ decreases their wealth and thus land prices. Thus, in this case,

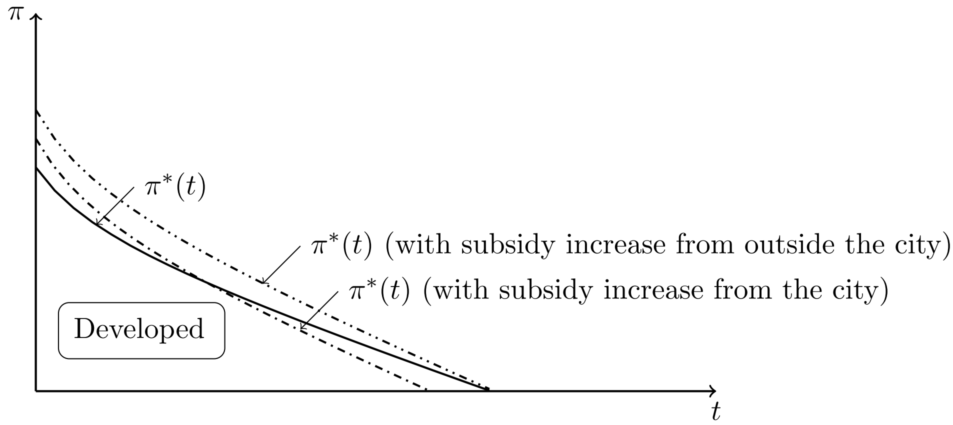

Figure 4: City boundaries and developed areas, as a function of transport cost t and probability π of natural hazard.

the increase of insurance subsidy leads to a land price increase in strongly risky areas and a land price decrease in weakly risky areas. If the households in the city do not bear the cost of insurance subsidization (i.e. $\overline{\tau} = 0$), an increase of λ does not have this negative effect on land prices. Thus, in this case, the increase of insurance subsidy leads to a general land price increase in the city because the increase of attractiveness of the city is not lowered by a tax on households in the city. Figure 4 illustrates the impact of a subsidy increase on the city boundaries for an "open city" in the case where the subsidy is borne by households in the city and in the case where the subsidy is not borne by households in the city. Besides, (18) states that the housing price $p_h(x)$ is modified through three channels while increasing insurance subsidy (λ): the land unit price $p_l(x)$, the housing lot size $s(x)$ and the building resilience $b(x)$. The direction of the impacts through the land unit price and the housing lot size depends on the location, similarly to these two variables. The impact through the building resilience decreases the housing unit price because the increase of insurance subsidy decreases the building resilience and thus its cost.

5 Conclusion

The paper has analyzed urban development choices in a city prone to natural disasters. It complements previous studies, in particular by including building resilience choices. Riskier areas are developed nearer to the city center than further away. Investment in building resilience leads to more compact cities. At a given distance to the city center, riskier areas have lower land prices and get lower household density and higher building resilience if insurance is not fully subsidized. Actuarially fair insurance leads households to optimally settle in space in terms of density and resilience. An increase of insurance subsidization leads to an increase of density in the riskiest areas of the city, in particular displacing inner boundaries towards riskier areas near the city center. Moreover an increase of insurance subsidization leads to a general decrease of building resilience in the city. To avoid excessive exposure to risk in the case of insurance subsidization, policy makers have to complement their policies by enforcing density and zoning restrictions as well as building codes. In this perspective, the present paper tells that, in the case of insurance subsidization, density and zoning restrictions have to be enforced at least in the riskiest areas of the city, in particular near the city center where land is attractive because of low transport costs. It also tells that, in the case of insurance subsidization, building codes should be generally enforced in the city for Pareto improvement.

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A Risk-prone city development

A.1 Characteristics of $\tilde{z}(.)$

The derivation of $v(\tilde{z}(s,v),s) = v$ relative to s gives:

$$\frac{\partial v}{\partial z}\frac{\partial \tilde{z}}{\partial s} + \frac{\partial v}{\partial s} = 0, \tag{20}$$

which can be rewritten:

$$\frac{\partial \tilde{z}}{\partial s} = -\frac{\partial_s v}{\partial_z v}. \tag{21}$$

Because $v(.)$ is increasing with z and s, $\tilde{z}(.)$ is decreasing with s. Besides, the derivation of (20) relative to s gives:

$$\frac{\partial^2 v}{\partial z^2}\left(\frac{\partial \tilde{z}}{\partial s}\right)^2 + \frac{\partial v}{\partial z}\frac{\partial^2 \tilde{z}}{\partial s^2} + 2\frac{\partial^2 v}{\partial z \partial s}\frac{\partial \tilde{z}}{\partial s} + \frac{\partial^2 v}{\partial s^2} = 0, \tag{22}$$

which can be rewritten with (21):

$$\frac{\partial v}{\partial z}\frac{\partial^2 \tilde{z}}{\partial s^2} = -\frac{\partial^2 v}{\partial z^2}\left(\frac{\partial_s v}{\partial_z v}\right)^2 + 2\frac{\partial^2 v}{\partial z \partial s}\frac{\partial_s v}{\partial_z v} - \frac{\partial^2 v}{\partial s^2}. \tag{23}$$

The term on the right-hand side of (23) is positive because $v(.)$ is concave and the determinant of the Hessian matrix of $v(.)$ is positive. Thus, $\frac{\partial^2 \tilde{z}}{\partial s^2}$ is positive and $\tilde{z}(.)$ is decreasing with s at a decreasing rate.

A.2 Derivation of (8) and (9)

With (21) and the expression (7) of $p_l(x)$, the first order conditions of (7) relative to $s(x)$ and $b(x)$ are respectively:

$$\frac{\frac{\partial_s v}{\partial_z v}\big(s(x),\tilde{z}(s(x),\overline{v})\big) - (1-\lambda)\pi(x)\frac{\partial l}{\partial s}\big(s(x),b(x)\big) - \frac{\partial c}{\partial s}\big(s(x),b(x)\big)}{s(x)} - \frac{p_l(x)}{s(x)} = 0, \tag{24}$$

$$-(1-\lambda)\pi(x)\frac{\partial l}{\partial b}\big(s(x),b(x)\big) - \frac{\partial c}{\partial b}\big(s(x),b(x)\big) = 0, \tag{25}$$

which respectively give (8) and (9).

A.3 Proof of proposition 1

As the first order conditions of (7) correspond to a maximum, the second order conditions of (7) are negative at the solutions $s(x)$ and $b(x)$. Thus, the following expressions which are called $A_1(x)$ and $A_2(x)$ are positive:

$$A_1(x) = \frac{\partial^2 \tilde{z}}{\partial s^2}\big(s(x),\overline{v}\big) + (1-\lambda)\pi(x)\frac{\partial^2 l}{\partial s^2}\big(s(x),b(x)\big) + \frac{\partial^2 c}{\partial s^2}\big(s(x),b(x)\big) \geq 0, \tag{26}$$

$$A_2(x) = (1-\lambda)\pi(x)\frac{\partial^2 l}{\partial b^2}\big(s(x), b(x)\big) + \frac{\partial^2 c}{\partial b^2}\big(s(x), b(x)\big) \geq 0. \tag{27}$$

With null cross derivation for $l(.)$ and $c(.)$ relative to their two arguments, the spatial derivation of (8) and (9) respectively gives:

$$A_1(x)\frac{ds}{d\vec{x}} = \frac{1}{s(x)}\frac{dt}{d\vec{x}} + (1-\lambda)\left(\frac{l\big(s(x), b(x)\big)}{s(x)} - \frac{\partial l}{\partial s}\big(s(x), b(x)\big)\right)\frac{d\pi}{d\vec{x}}, \tag{28}$$

$$A_2(x)\frac{db}{d\vec{x}} = -(1-\lambda)\frac{\partial l}{\partial b}\big(s(x), b(x)\big)\frac{d\pi}{d\vec{x}}, \tag{29}$$

which gives proposition 1.

B The impact of insurance subsidization

B.1 Optimal allocation

The first welfare theorem predicts the Pareto optimality with $\lambda = 0$ because efficient insurance markets would lead to actuarially fair insurance. For the formal proof, the optimal allocation with uniform welfare level $\overline{v}$ is obtained by minimizing the total expenditure of the city with N households:

$$\begin{aligned} \min_{s(.), b(.), n(.)} & \iint \Big(\tilde{z}\big(s(x), \overline{v}\big) + t(x) + \pi(x) l\big(s(x), b(x)\big) + c\big(s(x), b(x)\big)\Big) n(x) dx_1 dx_2 \\ & \text{s.t. } n(x)s(x) \leq 1, \forall x \\ & N = \iint n(x) dx_1 dx_2. \end{aligned} \tag{30}$$

The first order conditions give similar equations to the decentralized economy with $\lambda = 0$. Thus, the actuarially fair insurance policy (i.e. with $\lambda = 0$) implements the Pareto optimal allocation of resources.

B.2 Proof of proposition 5

The proof of proposition 5 is similar to the proof of proposition 1. The positive $A_1(x)$ and $A_2(x)$ are defined by (26) and (27). Contrary to proposition 1, the derivation of (8) and (9) relative to λ at a given location x do not have terms with derivatives of $t(x)$ and $\pi(x)$ but have terms with derivatives of $\overline{v}$ and $\overline{\tau}$. With null cross derivation for $l(.)$ and $c(.)$ relative to their two arguments, the derivation of (8) and (9) relative to λ at a given location x respectively gives:

$$s(x)A_1(x)\frac{ds(x)}{d\lambda} = \pi(x)s(x)\left(\frac{\partial l}{\partial s}(s(x), b(x)) - \frac{l(s(x), b(x))}{s(x)}\right) + \alpha(x)\frac{d\overline{v}}{d\lambda} + \frac{d\overline{\tau}}{d\lambda}, \tag{31}$$

$$A_2(x)\frac{db(x)}{d\lambda} = \pi(x)\frac{\partial l}{\partial b}(s(x), b(x)), \tag{32}$$

in which $\alpha(x) = \frac{\partial \tilde{z}}{\partial v}(s(x), \overline{v}) - s(x)\frac{\partial^2 \tilde{z}}{\partial v \partial s}(s(x), \overline{v})$. This gives proposition 5.