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# Optimal insurance for catastrophic risk: theory and application to nuclear corporate liability

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# Optimal insurance for catastrophic risk: theory and application to nuclear corporate liability

Alexis Louaas\* and Pierre Picard<sup>†</sup>

May 23, 2017

## Abstract

This paper analyzes the optimal insurance for low probability - high severity accidents, such as nuclear catastrophes, both from theoretical and applied standpoints. We show that the risk premium of such catastrophic events may be a non-negligible proportion of individuals' wealth when the index of absolute risk aversion is sufficiently large in the accident state, and we characterize the optimal asymptotic insurance coverage when the probability of the accident tends to zero. In the case of the limited liability of an industrial firm that may cause large scale damage, the limit corporate insurance contract corresponds to a straight deductible indemnification rule, in which victims are ranked according to the severity of their losses. As an application of these general principles, we consider the optimal corporate liability insurance for nuclear risk, in a setting where the risk is transferred to financial markets through catastrophe bonds. A model calibrated with French data allows us to estimate the optimal liability of a nuclear energy producer. This leads us to the conclusion that the lower limit adopted in 2004 through the revision of the Paris Convention is probably inferior to the socially optimal level.

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# 1 Introduction

What qualifies a low probability-high severity accident risk as a disaster risk? How should individuals and societies cover these risks? The present paper approaches these questions from theoretical and applied perspectives. Our motivation and ultimate objective is to analyze the case of nuclear accident risk.

We address the first question by characterizing individual preferences under which the risk premium may remain significant when potential losses are large, even when the loss probability is very small. The Arrow-Pratt formula provides a benchmark for small risks: the normalized risk premium (i.e. the risk premium per unit of variance) is approximately half of the index of absolute risk aversion, evaluated at the risk free reference point. In the case of a disaster risk, we show that the normalized risk premium has a lower bound which is half of a weighted average of absolute risk aversion values in the interval defined by the potential values of final wealth. In particular, under decreasing absolute risk aversion, a high absolute risk aversion (or, equivalently, a low risk tolerance) in the accident state may entail a large risk premium, even if the accident probability is very low. We then investigate the optimal insurance coverage of an individual who faces the risk of an accident with a very low probability, and we find that it converges to a limit when the accident probability goes to zero. This limit depends on the usual determinants of insurance demand: the insurance pricing rule and the individuals' wealth and degree of risk aversion.

In a second stage we consider the risk of an industrial accident, such as nuclear catastrophes, that may affect the entire population of a country. Should an accident occur, the firm has to indemnify the victims according to liability law, and it purchases insurance to prevent any insolvency. We characterize the indemnification rule that should be implemented by a utilitarian regulator. We show that, when the accident probability goes to zero, it converges toward a straight deductible indemnity schedule, capped by an upper limit. In particular, the optimal coverage crucially depends on the cost of capital that has to be levied to sustain the indemnification mechanism.

Finally, as an application of these theoretical principles, we consider the case of nuclear risk. Using studies conducted by experts in safety for a nuclear reactor in France, we calibrate a model of collective insurance choice and characterize the optimal level of coverage for the victims of a large scale nuclear accident. In particular, we use data from the catastrophe bond (henceforth cat bond) market to infer the premium that would be required by investors to set up an insurance deal for nuclear accidents. Our simulations suggest that the French nuclear liability law could be more ambitious than it currently is,

even after the 2004 revision of the international Paris Convention.

Our model establishes a bridge between the classical literature on optimal insurance coverage initiated by Mossin (1968), Arrow (1974) and Raviv (1979), the more recent works on cat bond pricing such as Braun (2015) and Perez and Carayannopoulos (2015) and the studies on risk coverage through cat bonds following Froot (2001).<sup>1</sup>

The particular role played by the index of absolute risk aversion in the worst case scenario also echoes the discussion of Weitzman's (2009) VSL-like (Value of a Statistical Life) parameter, that determines how tail risk affects the valuation of risk mitigating measures. Weitzman (2009) and Ikefuji et al. (2015) show, from a theoretical perspective, that the calibration of this VSL-like parameter can play a crucial role in determining the social cost of a heavy tail risk. Under given combinations of assumptions on the loss distribution and utility functions, extreme results are to be expected. For example, with a CRRA utility function, the marginal utility may be arbitrarily large in the worst state. This may prevent convergence of the stochastic discount factor if the probability distribution function places enough mass on the realization of the catastrophic events. As a result, arbitrarily large levels of risk mitigation can be rationalized under traditional methods of cost-benefit analysis. This result, known as the dismal theorem, warns against the sensitivity of cost-benefit analysis to the joint calibration of the utility function and the probability density function. Our work identifies and explores the area located between the dismal theorem world and the risk neutral world. We highlight the conditions under which even the most remote risks represent a sufficient threat for agents to undertake costly mitigating actions.

Even though our goal is not to explain asset prices, our work can also be related to the disaster equity premium literature initiated by Rietz (1988) and pursued by Barro (2006, 2009) and Gabaix (2009). Our contribution to this literature is to highlight the conditions under which a risk remains significant despite its low probability, and is therefore susceptible to yield significant aggregate effects on the risk premium.

The paper is organized as follows. Section 2 analyzes the risk premium and the insurance demand for a low-probability high-severity accident from the perspective of a risk averse individual. Section 3 characterizes the optimal corporate liability insurance when a large scale industrial accident may affect

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<sup>1</sup>Borensztein et al. (2017) also study the welfare gains induced by using cat bonds against natural disasters in developing countries, and Härdle and Cabrera (2010) calibrate cat bonds to cover earthquake risk in Mexico. An interesting parallel can also be drawn with Coval et al. (2009), who characterize senior structured fixed income securities as "economic catastrophe bonds", given that these assets only default under very adverse economic conditions.

the whole population of a country. Section 4 illustrates these general results through a calibrated model of nuclear catastrophe coverage where insurance risk is transferred to financial markets through catastrophe bonds. Section 5 concludes, Section 6 is an appendix that contains proofs and tables.

## 2 Risk premium and insurance demand for catastrophic risks

### 2.1 The risk premium of low-probability and high-severity risks

Consider an expected utility risk-averse individual with a von Neumann-Morgenstern utility function  $u(x)$  such that  $u' > 0$  and  $u'' < 0$ , where  $x$  is the individual's wealth. Let  $A(x) = -u''(x)/u'(x)$  and  $T(x) = 1/A(x)$  be her indices of absolute risk aversion and of risk tolerance, respectively. She holds an initial wealth  $w$ , and she is facing the risk of a loss  $L < w$  with probability  $p$ . Thus  $m(p, L) = pL$  and  $\sigma^2(p, L) = p(1 - p)L^2$  are the expected loss and the variance of the loss, respectively. The certainty equivalent  $C(p, L)$  of this lottery is defined by

$$u(w - C) = (1 - p)u(w) + pu(w - L).$$

We also denote

$$\theta(p, L) \equiv \frac{C(p, L) - m(p, L)}{\sigma^2(p, L)},$$

the normalized risk premium, that is the risk premium per unit of variance of the risk. Straightforward calculations give

$$\begin{aligned} C'_p(p, L) &= \frac{u(w) - u(w - L)}{u'(w - C)} > 0, \\ C''_{p^2}(p, L) &= -C'_p(p, L)^2 A(w - C) < 0. \end{aligned}$$

Thus,  $C(p, L)$  is increasing and concave with respect to  $p$ , and of course we have  $C(0, L) = 0$ .

Put informally, the risk  $(p, L)$  may be considered catastrophic for the individual if  $C(p, L)$  is non-negligible, for instance as a proportion of her initial wealth  $w$ , although  $p$  is small or even very small. Obviously, this may occur if  $C'_p(0, L)$  is large. We have

$$C'_p(0, L) = \frac{u(w) - u(w - L)}{u'(w)}. \quad (1)$$

Using l'Hôpital's Rule gives

$$\theta(0, L) \equiv \lim_{p \rightarrow 0} \theta(p, L) = \frac{C'_p(0, L) - L}{L^2}. \quad (2)$$

Thus, for  $L$  given, the larger  $C'_p(0, L)$ , the larger the normalized risk premium when  $p$  goes to zero. In other words, analyzing the determinants of  $\theta(0, L)$  is an intermediate step to understanding why  $C'_p(0, L)$  may be large.

We know from the Arrow-Pratt approximation that the risk premium of low-severity risks per unit of variance is proportional to the index of absolute risk aversion. Indeed, we have

$$\lim_{L \rightarrow 0} \theta(p, L) = \frac{A(w)}{2} \text{ for all } p \in (0, 1),$$

which of course also holds when  $p$  goes to 0, that is

$$\lim_{L \rightarrow 0} \theta(0, L) = \frac{A(w)}{2}.$$

When  $L$  is large, it is intuitive that the size of the risk premium depends on function  $A(x)$  not only in the neighborhood of  $x = w$ , but over the whole interval  $[w - L, L]$ . Proposition 1 and its corollaries confirm this intuition. Proposition 1 provides an exact formula for  $\theta(0, L)$  which is a weighted average of  $A(x) \exp\{\int_x^w A(t)dt\}/2$  when  $x$  is in  $[w - L, w]$ . Corollary 1 directly deduces a lower bound for  $\theta(0, L)$ , and Corollary 2 considers the case where  $L = w$  and the index of relative risk aversion  $R(x)$  is larger or equal to one.<sup>2</sup> In this case, the lower bound of  $\theta(0, L)$  is the (non-weighted) average of  $A(x)$  when  $x \in [0, w]$ .

**Proposition 1** *For all  $L > 0$ , we have*

$$\theta(0, L) = \frac{1}{2} \int_{w-L}^w [k(x)A(x) \exp\{\int_x^w A(t)dt\}]dx$$

where  $k(x) = 2[x - (w - L)]/L^2$  and

$$\int_{w-L}^w k(x)dx = 1.$$

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<sup>2</sup>Most empirical studies usually lead to values of  $R(x)$  that are larger (and sometimes much larger) than one, and thus the assumption made in Corollary 2 does not seem to be, in practice, very restrictive.

**Corollary 1** For all  $L > 0$ , we have

$$\theta(0, L) > \frac{1}{2} \int_{w-L}^w k(x)A(x)dx.$$

**Corollary 2** If  $L = w$ ,  $R(x) \equiv xA(x) \geq 1$  for all  $x$  and  $u(0) \in \mathbb{R}$  then

$$\theta(0, L) > \frac{1}{2w} \int_0^w A(x)dx.$$

With the DARA case in mind, Proposition 1 and its corollaries suggest that  $\theta(0, L)$  may be large if  $A(x)$  is large when  $x$  goes to  $w - L$ . A simple example, illustrated in Figure 1, is as follows. Assume  $L = w$  and

$$u(x) = \begin{cases} 1 - \exp(-ax) & \text{if } x \leq \hat{x} \\ b + cx & \text{if } x > \hat{x} \end{cases}$$

where  $b = 1 - w \exp(-a\hat{x})/(w - \hat{x})$  and  $c = \exp(-a\hat{x})/(w - \hat{x})$ , and  $\hat{x}$  is a fixed parameter such that  $0 < \hat{x} < w$ . Thus  $u(0) = 0$ ,  $u(w) = 1$  and  $A(x) = a$  if  $x \leq \hat{x}$  and  $A(x) = 0$  if  $x > \hat{x}$ .<sup>3</sup> When  $a$  is increasing (with a given value of  $\hat{x}$ ), the individual becomes more risk averse in the neighborhood of the bad outcome  $x = 0$ , with unchanged normalization  $u(0) = 0$ ,  $u(w) = 1$ . We then have  $C'_p(0, L) = 1/c = (w - \hat{x}) \exp(a\hat{x})$  and thus  $C'_p(0, L)$  is increasing with  $a$  and goes to infinity when  $a$  goes to infinity. Since  $\hat{x}$  is arbitrarily small, we learn from this example that  $C'_p(0, L)$  may be large if the individual is highly risk averse in the neighborhood of the loss state  $x = w - L$ , or equivalently if her risk tolerance is very small around this state.

Symmetrically, Proposition 2 shows that, under non-increasing absolute risk aversion, the normalized risk premium  $\theta(p, L)$  may be large when  $p$  is close to zero only if  $A(w - L)$  is very large, that is, only when the individual's risk tolerance is very small in the accident state.

**Proposition 2** Assume  $R(x) \equiv xA(x) \leq \bar{\gamma}$  for all  $x \in [w - L, w]$ . Then, under non-increasing absolute risk aversion, we have

$$\theta(0, L) < \frac{(\bar{\gamma} + 1)A(w - L)}{2},$$

and

$$C(p, L) < pL \left[ 1 + \frac{(\bar{\gamma} + 1)A(w - L)}{2} L \right].$$

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<sup>3</sup> $u(x)$  is not strictly concave since  $u''(x) = 0$  if  $x > \hat{x}$ , but this is for simplicity.

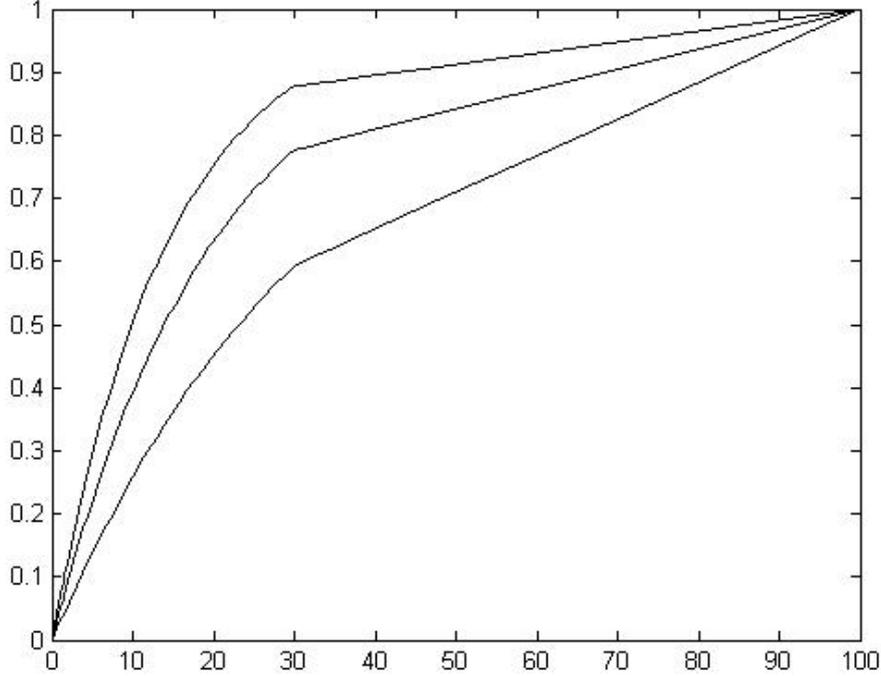


Figure 1:  $\hat{x} = 30$ ,  $w = 100$

Proposition 2 provides upper bounds for the normalized risk premium  $\theta(0, L)$  and for the certainty equivalent  $C(p, L)$  when the individual displays non-increasing risk aversion.  $\bar{\gamma}$  is an upper bound for the index of relative risk aversion  $R(x)$  when  $x$  is in the interval  $[w - L, w]$ . The upper bound of  $\theta(0, L)$  is proportional to  $A(w - L)$ , which is the index of absolute risk aversion in the loss state. Consequently,  $C(p, L)$  may be non-negligible when  $p$  is very small, say as a proportion of loss  $L$ , only if  $A(w - L)$  is large. On the contrary, assume  $A(w - L) = A(w)$ , i.e., the index of absolute risk aversion remains constant in  $[w - L, w]$ . In that case, we would have  $R(x) < R(w)$  for all  $x < w$ , and thus  $\bar{\gamma} = R(w)$ , which implies

$$C(p, L) < pL \left[ 1 + \frac{R(w)}{2} + \frac{R(w)^2}{2} \right].$$

Assuming  $R(w) = 2$  or  $3$  would give  $C(p, L) < 4pL$  or  $C(p, L) < 7pL$ , respectively. Thus, if  $p$  is very small, then  $C(p, L)/L$  is very small.<sup>4</sup>

<sup>4</sup>For the sake of numerical illustration, consider the case of a large scale nuclear disaster

Thus, under non-increasing absolute risk aversion, we may conclude that the risk premium of low-probability high-severity accidents may be non-negligible (and thus that the coverage of such a risk is a relevant issue) if and only if the risk tolerance is very low in such catastrophic cases.

*CRRA* preferences are an instance of such a case with  $T(x) = \gamma x$ , where  $\gamma$  is the index of relative risk aversion. We then have  $T(x) \rightarrow 0$  and  $A(x) \rightarrow \infty$  when  $x \rightarrow 0$ . However, *CRRA* preferences are not very satisfactory from a theoretical standpoint, since the utility is not defined when wealth is nil. This corresponds to discontinuous preferences in which any lottery with zero probability for the zero wealth state is preferred to any lottery with a positive probability for this state. If preferences are of the *HARA* type, then risk tolerance is a linear function of wealth, and we may write  $T(x) = a + bx$ , with  $a > 0$  and  $0 < b < 1$ . In such a case, we have  $A'(x) < 0$ ,  $A(0) = 1/a$  and  $R(x) > 1$ . In particular, the individual's absolute risk aversion index is decreasing but upper bounded. A straightforward calculation then gives

$$\frac{1}{2w} \int_0^w A(x) dx = \frac{1}{2bw} \ln \left( 1 + \frac{bw}{a} \right),$$

and thus, Corollary 2 shows that for all  $M > 0$ , we have  $\theta(0, L) > M$  if

$$a < \frac{bw}{\exp(2bwM) - 1}.$$

The right-hand side of the previous inequality is positive and decreasing in  $b$  and  $M$ . Thus,  $\theta(0, L)$  is arbitrarily large if  $a = T(0)$  is small enough and/or if  $b = T'(x)$  is small enough. In words, the risk tolerance should be low in the neighborhood of the catastrophic state  $x = 0$  for the normalized risk premium  $\theta(0, L)$  to be large.

Proposition 3 establishes a sufficient condition under which  $\theta(0, L)$  is (arbitrarily) large when the individual is sufficiently risk averse (or, equivalently, when her risk tolerance is sufficiently low) in the catastrophic loss state.

**Proposition 3** *Assume  $T(x) \equiv t(x, \varepsilon)$ , with  $\varepsilon > 0$ ,  $t(w - L, 0) = t'_x(w - L, 0) = t''_{xx}(w - L, 0) = 0$  and  $t'_x(x) > 0$  for  $x > w - L$ . Then for all  $M > 0$ ,  $\theta(0, L) > M$  if  $\varepsilon$  is small enough.*

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that may occur with probability  $p = 10^{-5}$ , with total losses of \$100b evenly spread among 1 million inhabitants (think of people living in the neighborhood of the nuclear plant). In the case of an accident, each inhabitant would suffer a loss  $L = \$100,000$ , with expected loss  $pL$  equal to \$1, and risk premium equal to \$4 or \$7, which would be negligible, say as a proportion of their annual electricity expenses. Postulating larger but still realistic values of the index of relative risk aversion would not substantially affect this conclusion.

In Proposition 3, it is assumed that the risk tolerance increases slowly (less than degree-two polynomials) when wealth increases in the neighbourhood of  $w - L$ . In such a setting, the normalized risk premium may be arbitrarily large if the risk tolerance in the loss state is small enough.

## 2.2 Insurance demand for catastrophic risks

We now assume that the individual can purchase insurance for a low-probability high-severity risk  $(p, L)$ . Insurance contracts specify the indemnity  $I$  in the case of an accident, i.e., when the individual suffers a loss  $L$ , and the premium  $P$  to be paid to the insurer, with  $P = (1 + \lambda)pI$ , where  $\lambda > 0$  is the loading factor such that  $p(1 + \lambda) < 1$ . The policyholder then faces the lottery  $(w_1, w_2)$ , with corresponding probabilities  $1 - p$  and  $p$ , where  $w_1$  and  $w_2$  denote respectively the wealth in the no-loss and loss states, with  $w_1 = w - P$  and  $w_2 = w - P - L + I$ . A straightforward calculation shows that feasible lotteries are defined by

$$[1 - p(1 + \lambda)]w_1 + (1 + \lambda)pw_2 = w - (1 + \lambda)pL, \quad (3)$$

with

$$w_2 - w_1 + L \geq 0, \quad (4)$$

for the sign condition  $I \geq 0$  to be satisfied. The optimal lottery maximizes the individual's expected utility

$$(1 - p)u(w_1) + pu(w_2),$$

in the set of feasible lotteries. It is such that the marginal rate of substitution  $-dw_2/dw_1|_{Eu=ct.} = (1 - p)u'(w_1)/pu'(w_2)$  is equal to the slope (in absolute value) of the feasible lotteries lines, that is

$$(1 - p)(1 + \lambda)u'(w_1) = [1 - (1 + \lambda)p]u'(w_2). \quad (5)$$

Figure 2 shows the locus of optimal lotteries in the  $(w_1, w_2)$  plane when  $p$  changes. Point A represents the situation with no insurance, and point B represents the optimal lottery when  $p$  goes to zero.

Let  $w_1(p, L)$ ,  $w_2(p, L)$  denote the optimal state-contingent wealth levels when  $I > 0$ , that is, when  $\lambda$  is not too large. Let us also denote

$$\begin{aligned} w_1^*(L) &\equiv \lim_{p \rightarrow 0} w_1(p, L) = w, \\ w_2^*(L) &\equiv \lim_{p \rightarrow 0} w_2(p, L), \end{aligned}$$

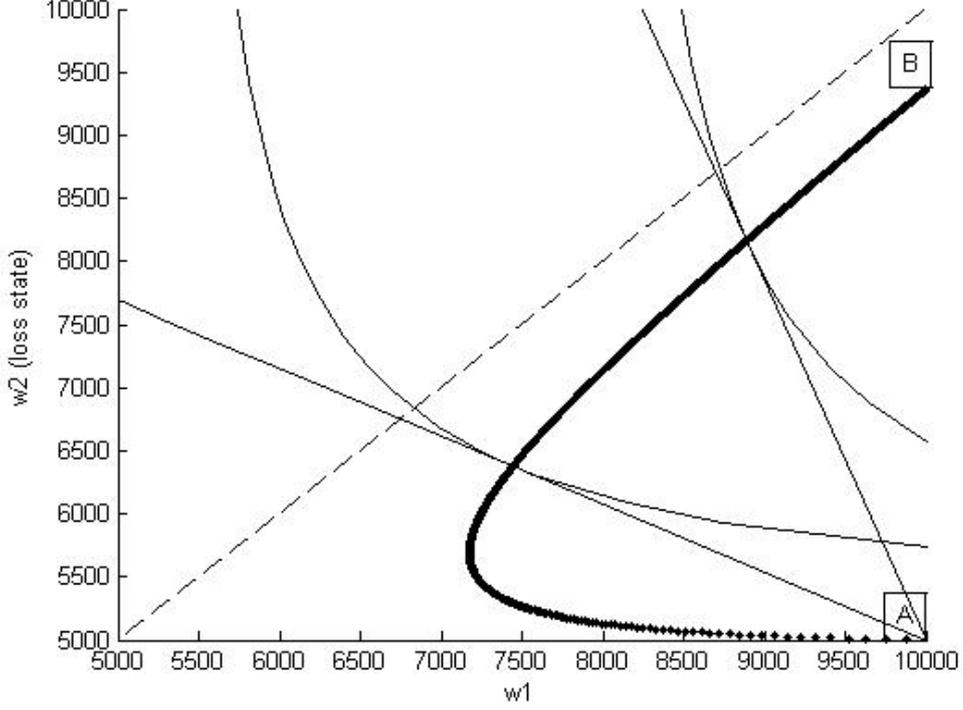


Figure 2:  $w = 10000$ ,  $L = 5000$ ,  $u(x) = -\frac{x^{-3}}{3}$

with

$$u'(w_2^*(L)) = (1 + \lambda)u'(w), \quad (6)$$

which implies  $w_2^*(L) < w = w_1^*(L)$ . Thus, when  $p$  goes to 0, the optimal insurance contract  $(P, I)$  goes to a limit  $(P^*, I^*)$ , with  $P^* = 0$  and  $I^* = w_2^*(L) + L - w_1^*(L) < L$ . When  $p$  is positive but close to 0, we still have  $I < L$  and  $P = (1 + \lambda)pI \simeq (1 + \lambda)pI^*$ . Since  $w_2^*(L) = w - L + I^*$ , (6) gives

$$u'(w - L + I^*) = (1 + \lambda)u'(w),$$

or

$$I^* = u'^{-1}((1 + \lambda)u'(w)) - w + L,$$

and thus  $I^*$  is decreasing with  $\lambda$ . The previous reasoning is valid only if  $I^* > 0$ , which holds if

$$u'(w - L) > (1 + \lambda)u'(w),$$

that is, if the loading factor  $\lambda$  is not too large or  $L$  is sufficiently large.

**Lemma 1**  $\lambda \leq \theta(0, L)L$  is a sufficient condition for  $I^* > 0$ .

Hence, the agent will be willing to buy a positive (and potentially large) amount of coverage if the normalized risk premium  $\theta(0, L)$  is larger than the ratio of the loading factor  $\lambda$  divided by the size of the loss  $L$ .

We may characterize the effect of a change in  $L$  and/or  $w$  on optimal insurance coverage. An increase  $dL > 0$  for  $w$  given induces an equivalent increase  $dI^* = dL$ . A simultaneous increase  $dw = dL > 0$  induces an increase  $dI^* > 0$  in coverage, while an increase in wealth with unchanged loss  $dw > 0, dL = 0$  entails a decrease in optimal coverage  $dI^* < 0$  under DARA references, i.e. when  $A' < 0$ . Of course, there is nothing astonishing here. These are standard comparative statics results, which are extended to the asymptotic characterization of catastrophic risk optimal insurance. They are summarized in Proposition 4.

**Proposition 4** *When  $p$  goes to 0, the optimal insurance coverage  $I$  goes to a limit  $I^*$ , and when  $p$  is close to 0, coverage  $I$  and premium  $P$  are close to  $I^*$  and  $(1 + \lambda)pI^*$ , respectively.  $I^*$  is lower than  $L$ , and is decreasing with  $\lambda$ . A simultaneous uniform increase in  $L$  and  $w$  induces an increase in  $I$  and  $P$ . Under DARA, an increase in  $w$  with  $L$  unchanged induces a decrease in  $I$  and  $P$ .*

## 3 Optimal catastrophic risk coverage for a population

### 3.1 Optimal contract

With the case of nuclear accident risk in mind, we now consider a population of individuals who face the risk of a catastrophic event (called "the accident") caused by a firm. Such an accident may affect the individuals differently, according to their risk exposure and also to their good or bad luck. The population has unit mass, and is composed of  $n$  groups or types indexed by  $i = 1, \dots, n$ , and a proportion  $\alpha_i$  of the population belongs to group  $i$ , with  $\alpha_1 + \alpha_2 + \dots + \alpha_n = 1$ . In the case of a nuclear accident caused by a given reactor, the groups correspond to various locations that may be more or less distant from the nuclear power plant. The accident occurs with probability  $\pi$ . In the case of an accident, a proportion  $q_i \in [0, 1]$  of type  $i$  individuals suffers damage, with financial damage  $\tilde{x}_i$  for each individual in this subgroup of victims.  $\tilde{x}_i$  is a random variable, whose realization is denoted  $x_i$ , and which is distributed over the interval  $[0, \bar{x}_i]$  with c.d.f.  $F_i(x_i)$  and density

$f_i(x_i) = F'_i(x_i)$ . The random variables  $\tilde{x}_i$  are independently distributed among type  $i$  individuals. Thus, we assume that in group  $i$  the victims are randomly drawn with probability  $q_i$ , and the law of large numbers guarantees that the proportion of affected individuals is equal to  $q_i$ , while their damage is independently distributed. The total cost of an accident is equal to

$$\sum_{i=1}^n \alpha_i q_i \left[ \int_0^{\bar{x}_i} x_i f(x_i) dx_i \right] = \sum_{i=1}^n \alpha_i q_i E \tilde{x}_i.$$

Under our assumptions, this total cost is given, but the distribution of loss between members of each group is random.

Each type  $i$  individual is covered by an insurance contract that specifies an indemnity  $I_i(x_i) \geq 0$  for all  $x_i$  in  $[0, \bar{x}_i]$ . This insurance coverage is taken out by the firm at price  $P$ . Once again, with the nuclear liability law in mind, we assume that the firm has to indemnify the victims according to the legal rule  $I_i(x_i)$  and also - in order to prevent any bankruptcy risk - that it has to purchase insurance to cover its liability. Thus,  $I_i(x_i)$  is at the same time the payment by the firm to type  $i$  individuals and the transfer from the insurer to the firm. The firm pays a premium  $P$  per individual, and this premium is passed on to the prices of the firm's product (say, on to the consumers' electricity bills). We assume that all consumers purchase the same quantity of the firm's products, and thus it is as if the insurance premium were paid by the individuals themselves.

Assume that the insurer allocates an amount of capital per individual  $K$  in order to pay indemnities, should an accident occur. The usual mutualization mechanism cannot be effective in the case of a small probability - large severity risk, and some alternative risk transfer is required. A simple approach (at least from a conceptual standpoint) consists in the insurer issuing a cat bond with par value  $K$ . The cat bond will pay some return (a spread above the risk-free rate of return), and will be reimbursed to investors only if no accident occurs. Otherwise, the cat bond will default, and its proceeds will be used to cover the claims for victims' compensation.<sup>5</sup>

We know from the law of large numbers that the average indemnity paid

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<sup>5</sup>In practice, a Special Purpose Vehicle (SPV) is created by the sponsor (here, the firm) as a legal entity able to host the cat bond. This SPV acts as an insurer or reinsurer with respect to the sponsor. It issues the bond, delivered to the investors in exchange for the principal payment, which entitles them to a regular coupon. Upon the occurrence of a contractually defined event, called the trigger, the bond defaults and the sponsor gets to keep the principal. Cat bonds are used by insurers and reinsurers to hedge against large losses among their portfolios of insured people, and by large corporations to cover catastrophic events.

to type  $i$  victims in the case of an accident is

$$\int_0^{\bar{x}_i} I_i(x_i) f_i(x_i) dx_i,$$

and thus the total indemnity payment can be financed if

$$K = (1 + \lambda) \sum_{i=1}^n \alpha_i q_i \int_0^{\bar{x}_i} I_i(x_i) f_i(x_i) dx_i,$$

where  $\lambda$  is a loading factor that represents the claim handling costs that the insurer faces beyond the indemnification costs. This cost of capital is covered by the premiums raised by the insurer, so we have

$$P = c(\pi, K)$$

with capital cost  $c(\pi, K)$  twice continuously differentiable,  $c'_K > 0$ ,  $c \rightarrow c_0^* \geq 0$  and  $c'_K \rightarrow 0$  when  $\pi \rightarrow 0$ ,  $c'_\pi > 0$ ,  $c''_{K^2} \geq 0$  and  $c''_{\pi K} \geq 1$ .<sup>6</sup>

Let  $w_1$  and  $w_{2i}(x_i)$  be the wealth of a type  $i$  individual if she is not affected by an accident (which occurs with probability  $1 - \pi q_i$ ), and if she is affected with loss  $x_i$  (which occurs with probability  $\pi q_i$  and conditional loss density  $f_i(x_i)$ ), respectively. We have

$$\begin{aligned} w_1 &= w - P, \\ w_{2i}(x_i) &= w - P - x_i + I_i(x_i). \end{aligned}$$

All individuals have the same initial wealth  $w$  and the same risk preferences represented by utility function  $u$ , with  $u' > 0$ ,  $u'' < 0$ .

Let  $C_i$  be the certainty equivalent loss of type  $i$  individuals. The set of feasible allocations  $\{w_1, w_{21}(x_1), \dots, w_{2n}(x_n), C_1, \dots, C_n, K\}$  is defined by

$$u(w - C_i) = (1 - \pi q_i)u(w_1) + \pi q_i \int_0^{\bar{x}_i} u(w_{2i}(x_i)) f_i(x_i) dx_i, \quad (7)$$

$$w_{2i}(x_i) - w_1 + x_i \geq 0 \text{ for all } i = 1, \dots, n, \quad (8)$$

$$K = (1 + \lambda) \sum_{i=1}^n \alpha_i q_i \int_0^{\bar{x}_i} I_i(x_i) f_i(x_i) dx_i, \quad (9)$$

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<sup>6</sup>If capital were levied through a cat bond, then  $c(K, \pi)/K$  would be the spread over LIBOR, i.e. the compensation per euro required by investors for running the risk of losing their capital with probability  $\pi$ . Under a zero risk-free interest rate, a risk neutral investor would require  $c(\pi, K) = \pi K$  to accept this risk. Note that we may have  $c(0, K) > 0$  if levying capital  $K$  induces fixed costs. See Section 4 for further developments.

$$w_1 = w - c(\pi, K). \quad (10)$$

Equation (7) defines  $C_i$  and equation (8) is a sign constraint for the insurance coverage. (9) defines the capital required to pay indemnities, and (10) follows from  $w_1 = w - P$  and  $P = c(\pi, K)$ .

We consider a utilitarian regulator that designs the risk coverage mechanism in order to minimize the social cost of an accident, which is the weighted sum of certainty equivalent to individuals' losses. The corresponding optimization program is also a way of characterizing the Pareto optimal allocations when ex-ante transfers between groups are possible.<sup>7</sup> This may be written as minimizing

$$\sum_{i=1}^n \alpha_i C_i,$$

with respect to  $\{w_1, w_{21}(x_1), \dots, w_{2n}(x_n); C_1, C_2, \dots, C_n, K\}$ , subject to conditions (7),(8), (9) and (10). Proposition 5 characterizes the optimal solution of this problem when  $\pi$  goes to 0 and  $K > 0$ .

**Proposition 5** *When  $\pi$  goes to zero with  $K > 0$ , all the optimal indemnity schedules  $I_i(x_i)$  converge toward a common straight deductible indemnity schedule  $I^*(x_i) = \max(x_i - d^*, 0)$  and  $K$  converges toward  $K^*$  defined by*

$$u'(w - d^*) = (1 + \lambda)u'(w - c_0^*)c''_{\pi K}(0, K^*),$$

$$K^* = (1 + \lambda) \sum_{i=1}^n \alpha_i q_i \left[ \int_{d^*}^{\bar{x}_i} (x_i - d^*) f_i(x_i) dx_i \right],$$

where  $c_0^* = c(0, K^*)$ .

Proposition 5 shows that the optimal indemnity schedule for small  $\pi$  involves full coverage of the victims above a straight deductible  $d^*$  (the same for all individuals whatever their type).<sup>8</sup> This amounts to saying that the victims should be ranked in order of priority on the basis of their losses: the victims with loss  $x_i$  should receive an indemnity only if the victims with loss  $x'_i$  larger than  $x_i$  receive at least  $x'_i - x_i$ . This simple characterization of optimal indemnification will be used in the simulation conducted in Section

<sup>7</sup>See Proposition 6 in the appendix for details.

<sup>8</sup>The fact that the deductible does not depend on type  $i$  is true only asymptotically when  $\pi \rightarrow 0$ . Otherwise, the optimal indemnity schedule involves type-dependent deductibles  $d_i$ , with  $I_i(x_i) = \max\{x_i - d_i, 0\}$ . This is because lower deductibles would allow the regulator to transfer wealth from more to less risky types (say from the groups with  $q_i$  high to the groups with  $q_i$  low if the conditional distribution of losses  $F_i(x_i)$  is the same for all groups). This compensatory effect vanishes when  $\pi$  goes to 0.

4. As in the simple model of Section 2.1, we may derive comparative statics properties for the asymptotic deductible  $d^*$ . In particular, it is increasing in  $\lambda$  and, under *DARA* preferences, it is increasing in wealth.

More importantly, Proposition 5 shows how  $d^*$  and  $K^*$  are affected by the cost of capital. If the investors were risk neutral, we would have  $c(\pi, K) = \pi K$ , i.e. the cost of capital would just be equal to the risk premium that compensates for the expected loss due to the default. We would have  $c''_{\pi K}(\pi, K) = 1$  and, in such a case, the cost of capital would not affect the optimal indemnity schedule.

However, as we will see in more detail in Section 4 with the example of the cat bond market for low-probability triggers, because of the aversion of investors towards risk, or for other reasons, it is much more realistic to keep the cost of capital in a more general form  $c(\pi, K)$ . In that case the cost of capital does affect the optimal indemnity schedule as highlighted in Proposition 5.<sup>9</sup>

## 4 The nuclear corporate liability case

### 4.1 The cost of capital

Financial innovations, including catastrophe bonds, have been developed during the two last decades in order to transfer large scale catastrophic risks to financial markets.<sup>10</sup> Alternative Risk Transfer markets include several categories of securities, including catastrophe bonds, designed to transfer catastrophic risks to dedicated financial investors. Focusing attention on the cat bond market, we may write  $c(\pi, K) = s(\pi, K)K$ , where  $s(\pi, K)$  denotes the spread over LIBOR for a cat bond.

The empirical literature has developed a number of cat bond pricing mod-

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<sup>9</sup>Note that  $c''_{\pi, K}(0, K^*) = \lim_{\pi \rightarrow 0} (1 - \pi)c'_K(\pi, K^*)/\pi$  from L'hôpital's rule. Then, Proposition 5 yields, for  $\pi$  small enough

$$\frac{\pi u'(w - d^*)}{(1 - \pi)u'(w - c_0^*)} \approx (1 + \lambda)c'_K(\pi, K^*).$$

The left-hand side of this equality is the individual's marginal rate of substitution between the states where he receives an indemnity after an accident and where no accident occurs, respectively. The right-hand side is the marginal cost of capital needed to sustain the insurance coverage, inflated by the loading factor  $\lambda$ . Hence, the first condition in Proposition 5 may be interpreted as the equality between marginal willingness to pay and marginal cost of coverage. The second equation is just a rewriting of equation (9) for the straight indemnity schedule  $I^*(x_i)$ .

<sup>10</sup>See Barriau and Cummins (2013).

els, of which we present four examples in Appendix 6.7. However, these models suffer from a lack of theoretical foundations and they predict unrealistically high spreads for cat bonds with very low probability triggers.<sup>11</sup> We therefore develop in Appendix 6.2 a simple one factor cat bond pricing model with the following features. The representative investor is assumed to be risk averse. In addition to the compensation for his expected loss, he therefore demands a premium for the systemic component of the risk that is correlated with his own wealth. He also requires a compensation for the underwriting and verification costs induced by the cat bond transaction. Our predictions for low probability cat bonds will therefore lie between two extremes. Spreads will be lower than those predicted by the existing models, presented in Appendix 6.7, but higher than those predicted in a model with risk neutral investors and no fixed cost. Our pricing equation is as follows

$$s = \pi(1 + \mu)\mathbb{E}(\tilde{x}) + \eta\kappa(1 + \mu)\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K + \frac{D}{K}, \quad (11)$$

where  $\tilde{x}$  is the fraction of the cat bond's capital lost by investors when the cat bond defaults, and  $\eta$  and  $\kappa$  respectively reflect the representative investor's degree of risk aversion and the exposure of his own wealth to the catastrophe. Finally,  $\mu$  is a loading that covers the verification costs that the investor incurs when the cat bond defaults. While the first term of equation (11) is the spread that would be required by a risk neutral investor, the second term reflects a risk premium. Finally,  $D$  is a fixed underwriting cost independent of the size  $K$  or probability  $\pi$  of a capital loss.

Based on this model, we estimate the following regression

$$s_i = \beta_0\pi_i\mathbb{E}(\tilde{x}_i) + \beta_1\pi_i[\mathbb{E}(\tilde{x}_i^2) - \pi(\mathbb{E}\tilde{x}_i)^2]K_i + \frac{\beta_2(1 + \sum_i \gamma_i X_i)}{K_i} + \varepsilon_i, \quad (12)$$

by using information from the Artemis database on cat bond transactions.<sup>12</sup>  $s_i$  denotes the spread over LIBOR of cat bond  $i = 1, \dots, n$ . If  $\text{€}K_i$  is issued through cat bond  $i$ , the corresponding cost of capital incurred by the issuer is  $c_i = s_i K_i$ . The spread of cat bonds is explained by the expected loss per  $\text{€}$ ,  $\pi_i$ , conditional expected loss  $\mathbb{E}(\tilde{x}_i)$ , conditional expected loss squared,<sup>13</sup> capital issued  $K_i$ , and a vector of observable controls  $X_i$ , such as year of

<sup>11</sup>In these models, either  $s(0, K) > 0$  or  $c''(0, K) = +\infty$ , which makes risk coverage unattractive when  $\pi$  is very small.

<sup>12</sup><http://www.artemis.bm/>

<sup>13</sup>We only possess information on the expected value of the random variable  $\tilde{x}$ . We therefore compute  $\mathbb{E}(\tilde{x}_i^2)$  by making the assumption that  $\tilde{x}_i$  is uniformly distributed over an interval  $[a_i, 1]$ . We then calibrate  $a_i$  to match the expected value of the uniform distribution with its empirical counterpart  $\mathbb{E}(\tilde{x}_i)$ .

issuance and zone of peril covered that may affect the fixed underwriting cost.

The Artemis database contains more than two-hundred issues, some of which are divided into several tranches, characterized by different levels of risk, and therefore by different spreads. We have complete information for 185 of the most recent tranches, spanning an interval of six years (2011-2017), including the nature of perils, types of trigger, probability of a capital loss, expected loss,<sup>14</sup> spreads, and identity of sponsors. Relevant controls also include the year of issuance, the area of the peril covered, and the type of trigger.

Table 1 gives the main OLS estimates of this regression.<sup>15</sup> All parameters

	Estimates
$\beta_0$	1.4599*** (10.1094)
$\beta_1$	0.0028** (2.1819)
$\beta_2$	0.7490* (1.6634)
$R^2$	0.7860

Table 1: OLS estimates

are positive and consistent with theory. The first parameter  $\beta_0$  is estimated to be 1.4599, which indicates the presence of a loading  $\mu$  around forty-five percent. The second parameter  $\beta_1$ , that identifies the representative investor's risk aversion, is statistically significant at a 5% level.<sup>16</sup> The second term of the regression will play an important role, due to the large values taken by  $K$ , the cat bond's capital. Finally, the third parameter  $\beta_2$ , that captures the cat bond's fixed cost  $D$ , is estimated at 0.7490, which implies a fixed cost of €749,000. In the model of section 3, we have assumed  $\tilde{x} = 1$ , which, for our cat bond,<sup>17</sup> gives

$$c(\pi, K) = \beta_0\pi K + \beta_1\pi(1 - \pi)K^2 + \beta_2, \quad (13)$$

<sup>14</sup>The probability of a capital loss and the distribution of losses are evaluated by modeling companies independent from the sponsor and the investor.

<sup>15</sup>The full table, along with alternative specifications is reported in Appendix 6.6.

<sup>16</sup>The  $t$ -statistics, robust to heterogeneity, are reported in parenthesis below the estimates. \*\*\* (respectively \*\*) : significant at 1% (respectively 5%) level.

<sup>17</sup>For simplicity, we have designed a simple cat bond that defaults entirely in case of a catastrophe. In addition, the cat bond we are interested in belongs by design to the reference group of our econometrics specification, which is why the dummy controls do not appear in equation (13).

and in particular

$$c''_{K\pi}(0, K) = \beta_0 + 2\beta_1 K,$$

which is an ingredient of the formula provided in Proposition 5.

## 4.2 Individual lotteries

The probability distribution of losses due to a nuclear accident is difficult to assess because data is scarce, and we can only rely on the analysis developed by nuclear safety specialists. In particular, the Probabilistic Safety Assessment (PSA) studies seek to understand the odds and the stakes of a major accident along several dimensions: sanitary, environmental, economic, etc. Designed to improve prevention and the ex-post management of a crisis situation, they deliver, as a by-product, useful information about the probabilities of different scenarios, analyzed in detail in Dreicer et al. (1995) and Markandya (1995). Additional studies from international agencies, such as the French Institute for Radioprotection and Nuclear Safety (IRSN, 2013) and the Nuclear Energy Agency (NEA, 2000), also develop the methodology for estimating the costs associated with the various accident scenarios predicted by PSA studies.

As in Eeckhoudt et al. (2000), we make use of the aggregate information on costs and probabilities drawn from PSA studies to construct individual lotteries. We consider the risk associated with one major accident on the French territory.<sup>18</sup> The 58 French nuclear reactors are gathered into 19 power plants. Based on Eeckhoudt et al. (2000), we assume that 2 million people live around each power plant. Therefore 38 million people are located near a power plant (less than 100km) and 28 million people live further away. We index these two groups by  $i = 1, 2$ , with shares in the population  $\alpha_1 = 38/66$  and  $\alpha_2 = 28/66$ , respectively. We let  $\pi$  denote the probability that a major nuclear accident affects the territory. Most PSA studies provide very low estimates ranging from  $10^{-4}$  to  $10^{-9}$  per year and per reactor. We will use in our computation  $\pi = 58 * 10^{-5}$ ,<sup>19</sup> but since we approximate the optimal level of capital by its limit value, this calibration does not affect our results about the optimal coverage and deductible  $K^*$ ,  $d^*$ , but it does affect the premium  $P$ .

For any individual, the potential direct consequences of a nuclear accident may include financial losses, severe disease and death, and it is these losses

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<sup>18</sup>We use ST21 as a benchmark for the number of direct victims in our baseline scenario. The PSA studies referenced above provide the technical background on which ST21 relies.

<sup>19</sup>We neglect the possibility that accidents may occur simultaneously in several power plants.

that may be subject to compensation under corporate liability law. Other losses are supposed to be evenly spread over the whole population. When an accident occurs, an individual of group 1 has a probability  $1/19$  of living nearby the damaged power plant ( $< 100$  km),<sup>20</sup> in which case she can die, suffer a severe disease or a financial loss if she lives in the plume of radioactivity. With probability  $18/19$ , she lives away from the damaged power plant ( $\geq 100$  km), similar to a person from group 2, and can die or suffer a severe disease. The direct financial losses are incurred only by people in group 1, and may result from the impossibility to stay in a contaminated area.

We use figures similar to Eeckhoudt et al. (2000) to calibrate our baseline scenario. The number of direct victims in the baseline scenario (scenario 1) is summarized in Table 2.

Distance	Population	Financial loss	Death	Severe disease
$< 100$ km	2 million	10,000	500	1,000
$\geq 100$ km	64 million	0	3,000	6,000

Table 2: Population affected by direct losses in scenario 1

We assume that each person in the most exposed group (i.e., individuals from group 1, living within 100 km of a power plant) can potentially be in 6 distinct states (3 health states  $\times$  2 financial states)  $s_1 = 1, \dots, 6$ . Other individuals never incur the direct financial loss, so they can only be in three different health states  $s_2 = 1, \dots, 3$ . The lotteries associated with the baseline scenario are summarized in Tables 3 and 4. The initial wealth  $w$  is calibrated in euros, as the sum of the asset value currently held, plus the expected discounted future wealth of the average French citizen, which yields  $w = 875,310$  euros.<sup>21</sup>

People from group 1 die in states  $s_1 = 1$  and 2. They also suffer a financial loss in state  $s_1 = 1$  (and not in state  $s_1 = 2$ ). The worst possible case is monetized as a loss of a fraction  $1 - \theta$  of total wealth, where  $\theta$  can be interpreted as a bequest parameter. We choose the parameter  $\theta$  so as to match the value of a statistical life (VSL) recommended for cost-benefit analysis with a HARA utility function.<sup>22</sup> In particular, our baseline calibration with  $\theta = 10\%$  implies Values of a Statistical Life between 3 and 4 million euros, consistent with the estimates provided in Viscusi and Aldi's

<sup>20</sup>For simplicity, we assume that the 19 power plants have the same number of reactors. This approximation has very little impact on our results.

<sup>21</sup>The details of this calibration are presented in Appendix 6.4.

<sup>22</sup>The HARA utility function does not display a divergent index of absolute risk aversion when  $\theta$  goes to zero, except in the limit CRRA case. See equation 14 below.

(2003) meta-analysis and with Quinet (2013), which sets the standard for cost-benefit analysis in France. Finally Appendix 6.8 shows the robustness of our analysis to realistic changes in the parameter  $\theta$ .

People in state  $s_1 = 3$  do not die but they face the combined consequences of a severe disease and financial losses. In states  $s_1 = 4$  and  $s_1 = 5$ , they suffer either the severe disease or the financial shock, respectively, while in state  $s_1 = 6$  they do not incur direct losses. Table 3 presents these loss levels and the corresponding probability conditional on the occurrence of a nuclear accident.<sup>23</sup>

Concerning group 2, individuals die in state  $s_2 = 1$ , suffer a severe disease in state  $s_2 = 2$  and face no direct loss in state  $s_2 = 3$ .

State	Description of direct losses	Direct loss	Total loss	Probability
$s_1 = 1$	Death + financial loss	787,780	787,780	7.8947e-08
$s_1 = 2$	Death	717,780	719,220	5.7513e-05
$s_1 = 3$	Disease + financial loss	330,000	331,440	1.3158e-07
$s_1 = 4$	Disease	260,000	261,440	1.1500e-04
$s_1 = 5$	Financial loss	70,000	71,440	2.6297e-04
$s_1 = 6$	No direct loss	0	1,440	9.996e-01

Table 3: lotteries for type  $i = 1$

To these direct consequences, subject to compensation under corporate liability law, one must add more diffuse economic costs that are qualified as indirect costs in Schneider (1998) and subsequent studies. They are difficult to quantify and attribute to a given individual. Examples of such costs are: the loss of attractiveness of an impacted territory, loss in terms of image for the industrial sector, etc.<sup>24</sup> For simplicity, we assume that these costs are evenly shared by all individuals in the economy<sup>25</sup> and we keep the total cost of the accident fixed at 100 billion euros. In group  $i = 1$ , agents in state  $s_1 = 6$  only face the indirect loss from the accident. Total losses are obtained by adding direct and indirect losses.

<sup>23</sup>The state probabilities in Tables 3 and 4 are also conditional on belonging to group 1 and 2, respectively.

<sup>24</sup>Here we do not discuss the effect of the catastrophe on growth, as the literature has not reached a consensus on the growth effect of disasters. For example, Gignoux and Menéndez (2016) find a positive effect for the case of an earthquake in India, while Strobl (2012) finds a negative effect for the case of hurricanes in the Caribbean.

<sup>25</sup>We could also treat these indirect costs as uninsurable background risks. Under the risk vulnerability assumption, these background risks would increase the degree of risk aversion to insurable risks.

Alternative scenarios (scenario 2,3,4 and 5) are generated by multiplying the number of direct victims considered in Table 2 by 2,3,4 and 5, respectively, while reducing the value of indirect losses so as to keep the total cost fixed at 100 billion euros. Total direct losses range from 5 billion euros in scenario 1 to approximately 25 billion euros in scenario 5. Total indirect losses therefore vary between 75 and 95 billion euros. Because we assume that indirect losses are mutualized, they only marginally affect the optimal coverage level. Hence, as far as corporate liability is concerned, the assumption that total cost is 100 billion euros is innocuous.<sup>26</sup>

State	Description of direct losses	Direct loss	Total loss	Probability
$s_2 = 1$	Death	717,780	719,220	4.6875e-05
$s_2 = 2$	Disease	260,000	261,440	9.3750e-05
$s_2 = 3$	No direct loss	0	1,440	9.999e-01

Table 4: lotteries for type  $i = 2$

### 4.3 Optimal coverage

We postulate a harmonic absolute risk aversion (HARA) utility function

$$u(x) = \zeta \left( \eta + \frac{x}{\gamma} \right)^{1-\gamma},$$

whose domain is such that  $\eta + (x/\gamma) > 0$ , and with the condition  $\zeta(1-\gamma)/\gamma > 0$ , that guarantees that  $u(x)$  is increasing and concave. With affine risk tolerance  $T(x) = 1/A(x) = \eta + x/\gamma$ , the coefficient of relative risk aversion is written

$$R(x) = x \left( \eta + \frac{x}{\gamma} \right)^{-1}. \quad (14)$$

The HARA class nests the constant relative risk aversion (CRRA) case when  $\eta = 0$ , and the constant absolute risk aversion (CARA) case when  $\gamma \rightarrow +\infty$ . Except for the CARA and CRRA limit cases, HARA functions satisfy decreasing absolute risk aversion and increasing relative risk aversion. Studies on individual data, such as Levy (1994) and Szpiro (1986), have isolated a plausible range between 1 and 5 for the index of relative risk aversion. We therefore perform simulations over this plausible range of values.

The optimal values of the deductible and capital are deduced from Proposition 5 and Section (4.1). They are reported in Table 5 for a level of relative

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<sup>26</sup>In particular, assuming a total cost of 50 or 200 billion euros would not significantly modify our results.

$\underline{R}$	1		2	
Scenario	Cover	Welfare	Cover	Welfare
1	0.6873	0.0556	0.7529	0.0786
2	0.9646	0.0814	1.1021	0.1202
3	1.1454	0.0957	1.3495	0.1456
4	1.2775	0.1038	1.5429	0.1617
5	1.3801	0.1083	1.7016	0.1719

$\underline{R}$	1		2	
Scenario	Premium	Deductible	Premium	Deductible
1	2.1024	5.6828	2.3125	5.5385
2	3.0858	6.1324	3.6661	5.9813
3	3.8618	6.3530	4.8666	6.2034
4	4.4968	6.4899	5.9440	6.3441
5	5.0291	6.5850	6.9196	6.4437

Table 5: Optimal cover (in €billion), Welfare gain, Annual premium (in €millions), Deductible (in €hundreds of thousands),  $\bar{R} = 2$

risk aversion  $\bar{R} := R(w) = 2$ , which is our baseline assumption.<sup>27</sup> Since the relative risk aversion has two degrees of freedom in the HARA case, we let  $\underline{R} := R(w - L(s_1))$ , where  $L(s_1)$  is the loss incurred in state  $s_1$  by group 1 individuals, vary across columns.<sup>28</sup> The scenario considered varies across lines.

Optimal levels of coverage (in billion euros) and their associated welfare gains are read from the top panel of Table 5. Annual premiums (in millions of euros) and deductibles (in hundreds of thousands of euros) are read from the bottom panel. If we consider a baseline set of assumptions with scenario 1,  $\underline{R} = 2$  and  $\bar{R} = 2$  (i.e. the CRRA case), we find an optimal level of coverage  $K^*$  equal to €752.900 million, an associated welfare gain of 7.86%, a deductible of €553,850 per inhabitant, and an annual premium of €2.3125 million (just below 3.5 cents per person). This yields a spread  $s = 2.3125/752.900 = 0.31\%$  that is one order of magnitude above the spread that a risk neutral investor would require in the absence of underwriting costs. In principle, these fixed underwriting costs can be an issue for the insurability of low probability events, but in our setting they are divided among a large number of agents and therefore have a small impact on each agent.

<sup>27</sup>A wider set of assumptions, with an index of relative risk aversion  $\bar{R}$  varying from 1 to 5, is considered in Appendix 6.8.

<sup>28</sup>In other words,  $\bar{R}$  and  $\underline{R}$  denote the index of relative risk aversion, in the no accident state and in the worst case state, respectively.

Table 5 highlights the dependence of the coverage and annual premium on the catastrophe scenario. When  $\underline{R} = 1$  and  $\overline{R} = 2$ , multiplying the number of people in each category of loss by 5 (i.e. comparing scenario 1 and 5) induces an increase in cover by a factor 2 and 2.26, respectively. The fact that coverage increases at a slower pace than direct losses is an intuitive result that is due to the increasing marginal cost of capital.

The deductible varies between €553,850 and €658,500 in Table 5. This represents more than half of the individual's wealth, which implies that only people in the worst states ( $s_1 = 1, 2$  for group 1 and  $s_2 = 1$  for group 2) are indemnified. Table 5 also confirms the intuition that deductibles should decrease with risk aversion, but the effect is quantitatively limited. Finally, the deductible increases with the severity of the loss scenario, which reflects our previous remark on the effect of increasing marginal cost of capital on optimal coverage. As more capital is needed to compensate the victims with the largest losses, it is optimal to increase the deductible in order to avoid a sharp increase in premiums.

The welfare gain is computed as the reduction in the loss certainty equivalent induced by the cover in comparison with the case without any compensation.<sup>29</sup> The welfare gain is therefore estimated at least at 5.56% under scenario 1 with  $\overline{R} = 2$  and  $\underline{R} = 1$ . This means that the average monetary equivalent cost of the nuclear risk is lowered by 5.56% thanks to the indemnity schedule when  $K^* = €687.300$  million. Of course, welfare gains for group 2, taken separately, would be higher. Higher values for the coefficients of relative risk aversion, or a more pessimistic loss scenario would lead to much higher values of  $K^*$  and substantially higher welfare gains.

Note finally that in scenario 1,  $K^*$  is close to the lower bound of nuclear operator's liability adopted in 2004 through the revision of the Paris convention, which is €700 million for each plant. Under the most severe scenarios 2, ..., 5, this lower bound would be inferior to the socially optimal level. The fact that several other European countries<sup>30</sup> have set nuclear corporate liability at higher levels is coherent with such scenarios.

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<sup>29</sup>Since group 1 and group 2 do not face the same risk exposure, this reduction differs from one group to the other. The figure presented in Table 5 is an average of these two gains weighted by group size.

<sup>30</sup>Countries have their own legislation, in line with international conventions. For instance, in Germany, the nuclear corporate liability is set at €2.5 billion for each plant. This could be rationalized in our model with scenarios more severe than our scenario 5, or with higher levels of risk aversion, such as the ones considered in Appendix 6.8. Note that the Paris convention also specifies tranches of liability born by governments, so that total liability toward the victims are at least €1.5 billion.

## 5 Conclusion

The structural changes experienced in modern economies, whether associated with technological, environmental or financial transformations, have highlighted how contemporary societies are more and more subject to large scale uncertainties, including catastrophic risks. The purpose of this paper was to analyze the insurability of low probability-high severity events, simultaneously from the individual's standpoint and a public policy perspective.

We have shown that the risk premium of such catastrophic events can remain large when the accident probability is close to zero, if the index of absolute risk aversion is sufficiently large (or equivalently if risk tolerance is sufficiently low) in the accident state. In addition, the optimal indemnity converges to a positive limit that reflects both the individual's attitude toward risk and the cost of insurance. In the case of an industrial catastrophe that may affect the whole population of a country the insurability issue is linked to the corporate liability of the firm that may cause the accident. The asymptotic indemnity schedule is characterized by a straight deductible, common to all individuals.

Based on these results, we have analyzed the features of an optimal insurance scheme that covers the nuclear accident risk in which the risk is transferred to financial markets through cat bonds. Using recent cat bond data and safety studies on nuclear reactors allows us to compute the optimal level of coverage. Our results, calibrated with French data, suggest that the nuclear liability law could be more ambitious than it currently is, unlike in other countries, such as Germany, where this liability has been extended far beyond the requirements of international conventions.

Our analysis presents a certain number of limits that we shall now discuss. First, we implicitly assume that market insurance is the only tool available to deal with catastrophic risk. In practice, individuals and societies have other means at their disposal. The effect of self-insurance -a reduction in the size of the loss- and self-protection -a reduction in the loss probability- were studied in a seminal paper by Ehrlich and Becker (1972). In a two state model, they showed that if market insurance and self-insurance are substitutes, self-protection and market insurance can be complements. Most of our theoretical results, being of a qualitative nature, would be unaffected by introducing self-insurance and self-protection, but we leave for future research the analysis of the interaction between self-insurance, or self-protection, and market insurance for disaster risks.

Another strategy to deal with risk is inter-temporal smoothing. In a dynamic model with uninsurable risk, prudent agents save to constitute a buffer used in case of loss. If market insurance is sold at actuarially fair prices, ex-

pected utility maximizing agents should purchase full insurance and make no precautionary savings. However, insurance being a costly activity, positive loadings may lower the demand for market insurance, substituted for by precautionary savings. Gollier (2003) showed, with a calibrated example, that the demand for market insurance may become quite low whenever assets enable agents to transfer wealth across periods. However, his example only discusses the case of small losses. The strategy of substituting market insurance with precautionary savings would not be feasible at the levels of individual agents who risk up to their lives. It could more realistically be set up at the level of the nation. We could imagine, for example, that the government constitutes a fund dedicated to the indemnification of large catastrophes, or simply borrows money when a catastrophe occurs. This strategy raises difficult concerns of inter-generational risk sharing very much like those debated in the literature on the social cost of environmental damage. Borensztein et al. (2017) highlight an interesting feature of cat bonds: even in a framework where they can be substituted for with a precautionary fund, cat bonds may yield substantial gains. The purchase of insurance cover against catastrophes may indeed lower the risk of default and thereby improve the terms of credit for the financing of the government's normal activities.

A last limitation of our analysis is the fact that we considered a single aggregate loss scenario for our quantitative assessment of the optimal level of coverage against the risk of nuclear accident. While this was dictated by the format of the PSA studies used to calibrate our loss model, an extension to several scenarios is an interesting avenue for further research.

## 6 Appendix

### 6.1 Complement to section 3.1

Let us assume that the government can redistribute wealth between groups through ex ante lump sum transfers. We denote  $t_i$  the net transfer paid to each individual of group  $i$ , the government budget constraint being written as

$$\sum_{i=1}^n \alpha_i t_i = 0.$$

Now we have

$$\begin{aligned} w_1 &= w - P + t_i, \\ w_{2i}(x_i) &= w - P - x_i + I_i(x_i) + t_i. \end{aligned}$$

and the certainty equivalent loss incurred by type  $i$  individuals is still denoted by  $C_i$ , with

$$\begin{aligned} u(w - C_i + t_i) &= (1 - \pi q_i) u(w_1 + t_i) \\ &\quad + \pi q_i \int_0^{\bar{x}_i} u(w_{2i}(x_i) + t_i) f(x_i) dx_i. \end{aligned} \quad (15)$$

An allocation is written as  $\mathcal{A} = \{w_1, w_{21}(x_1), \dots, w_{2n}(x_n), C_1, \dots, C_n, t_1, \dots, t_n, K\}$ , and  $\mathcal{A}$  is feasible if (8), (9), (10) and (15) are satisfied.

**Definition 2**  $\mathcal{A}$  is Pareto-optimal if it is feasible and if there does not exist another feasible allocation  $\hat{\mathcal{A}} = \{\hat{w}_1, \hat{w}_{21}(x_1), \dots, \hat{w}_{2n}(x_n), \hat{C}_1, \dots, \hat{C}_n, \hat{t}_1, \dots, \hat{t}_n, \hat{K}\}$  such that  $\hat{C}_i - \hat{t}_i \leq C_i - t_i$  for all  $i = 1, \dots, n$ , with  $\hat{C}_{i_0} - \hat{t}_{i_0} < C_{i_0} - t_{i_0}$  for at least one group  $i_0$ .

**Proposition 6**  $\mathcal{A} = \{w_1, w_{21}(x_1), \dots, w_{2n}(x_n), C_1, \dots, C_n, t_1, \dots, t_n, K\}$  is a Pareto-optimal allocation if and only if it minimizes  $\sum_{i=1}^n \alpha_i C_i$  in the set of feasible allocations.

### 6.2 A cat bond pricing model

This section presents the cat bond pricing model. The cat bond is issued at  $t = 0$ . Part of its capital is used at time  $t = 0$  to pay the underwriting costs and the remainder constitutes the principal. At time  $t = 1$  the principal  $K$  is returned to the investor if the accident did not occur. In the opposite case, the cat bond defaults and the sponsor uses a fraction  $\tilde{x}$  of the capital to indemnify the victims. The remaining portion of capital is returned to

the investors. From the standpoint of the investor, the cat bond's payoff is therefore

$$\tilde{q} = \begin{cases} RK + [1 - (1 + \mu)\tilde{x}]K & \text{with probability } \pi, \\ (1 + R)K & \text{with probability } 1 - \pi. \end{cases}$$

In compensation for the option to default on the principal, the investors require a coupon of rate  $R = r + s$ , where  $r$  denotes the risk free rate and  $s$  denotes the spread. We let  $D/(1 + r)$  be the value of the underwriting costs (i.e.  $D$  is the corresponding value at time  $t = 1$ ), and  $\mu$  is a loading that covers the verification costs.

Let  $CE$  be the certainty equivalent of the cat bond payoff  $\tilde{q}$  to investors at time  $t = 1$ . Following the Consumption Capital Asset Pricing Model, we write

$$CE = \mathbb{E}\tilde{q} - \eta cov(\tilde{z}, \tilde{q}),$$

where  $\tilde{z}$  denotes the wealth of the representative investor at  $t = 1$ , and  $\eta$  reflects his risk aversion. There are two states: with probability  $\pi$ , the accident occurs, the cat bond defaults and investors suffer a loss  $(1 + \mu)\tilde{x}K$ ; with probability  $1 - \pi$ , the accident does not occur and the principal is returned to the investor. In both cases, the coupon  $RK$  is paid to the investor.<sup>31</sup> We assume that the representative investor bears a fraction  $\kappa$  of the underlying loss.<sup>32</sup> We therefore write

$$\tilde{z} = \begin{cases} w - \kappa K \tilde{x} & \text{with probability } \pi, \\ w & \text{with probability } 1 - \pi. \end{cases}$$

Thus

$$\begin{aligned} \mathbb{E}\tilde{q} &= [R + 1 - \pi(1 + \mu)\mathbb{E}(\tilde{x})]K, \\ cov(\tilde{z}, \tilde{q}) &= (1 + \mu)\kappa\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K^2. \end{aligned}$$

and

$$CE = [R + 1 - \pi(1 + \mu)\mathbb{E}(\tilde{x})]K - \eta(1 + \mu)\kappa\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K^2.$$

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<sup>31</sup>Hence we assume that default affects the repayment of the capital to the investor first. The coupon payment is affected only when the loss  $\tilde{x}$  is very large and  $1 - (1 + \mu)\tilde{x}$  becomes negative. This assumption is made for simplicity, but of course other definitions of cat bonds are possible.

<sup>32</sup>We do not restrict  $\kappa$  and will estimate it from the data. From a theoretical perspective, the precise value of  $\kappa$  depends on the identity of the representative investor. If the representative investor is not exposed to the underlying risk transferred by the cat bond, we should have  $\kappa \equiv 0$ .

Purchasing the cat bond is analogous to making an investment  $K$  with additional cost  $D/(1+r)$  at  $t=0$  and random payoff  $\tilde{q}$ , with certainty equivalent  $CE$ , at  $t=1$ . Thus, in the absence of arbitrage, we have

$$K + \frac{D}{1+r} = \frac{CE}{1+r},$$

which may be rewritten as

$$K(1+r) = [R+1 - \pi(1+\mu)\mathbb{E}(\tilde{x})]K - \eta(1+\mu)\kappa\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K^2 - D.$$

Let  $s = R - r$  be the spread over the risk-free rate. We obtain

$$s = \pi(1+\mu)\mathbb{E}(\tilde{x}) + \eta\kappa(1+\mu)\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K + \frac{D}{K}. \quad (16)$$

In order to estimate this equation on our data set, we assume each  $\tilde{x}_i$  is uniformly distributed in an interval  $[a_i, 1]$ . This enables us to find  $\mathbb{E}(\tilde{x}_i^2)$  which, in turn, leads to the regression performed in section 4.1.<sup>33</sup> We only have one loss scenario in our numerical analysis. Hence, the cat bond must completely default in case of accident, which implies that  $\mathbb{E}(\tilde{x}) = 1$  for our cat bond. The cost of capital  $c(\pi, K) \equiv s(\pi, K)K$  is therefore

$$c(\pi, K) = \pi(1+\mu)K + \eta\kappa(1+\mu)\pi(1-\pi)K^2 + D,$$

which is coherent with the assumptions used to derive Proposition 5.

## 6.3 Proofs

### Proof of Proposition 1

From equation (1), we have

$$C'_p(0, L) = \frac{u(w) - u(w-L)}{u'(w)} = \int_{w-L}^w \frac{u'(x)}{u'(w)} dx.$$

Since

$$u'(x) = u'(w) - \int_x^w u''(t) dt,$$

for all  $x \in [w-L, w]$ , we may write

$$\begin{aligned} C'_p(0, L) &= L - \int_{w-L}^w \left[ \int_x^w \frac{u''(t)}{u'(w)} dt \right] dx \\ &= L + \int_{w-L}^w \left[ \int_x^w A(t) \frac{u'(t)}{u'(w)} dt \right] dx, \end{aligned}$$

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<sup>33</sup>The Artemis data base provides  $\pi_i$  and  $\mathbb{E}\tilde{x}_i$  for each cat bond  $i$  in the sample. We deduce  $\mathbb{E}(\tilde{x}_i^2)$ .

and thus

$$\theta(0, L) = \frac{1}{L^2} \int_{w-L}^w \left[ \int_x^w A(t) \frac{u'(t)}{u'(w)} dt \right] dx.$$

Integrating by parts gives

$$\theta(0, L) = \frac{1}{2} \int_{w-L}^w k(x) A(x) \frac{u'(x)}{u'(w)} dx, \quad (17)$$

where  $k(x) = 2[x - (w - L)]/L^2$ , with

$$\int_{w-L}^w k(x) dx = 1.$$

In addition, we have

$$u'(x) = u'(w) \exp\left\{ \int_x^w A(x) dx \right\},$$

which completes the proof.

### **Proof of Corollary 2**

When  $L = w$ , we have

$$\theta(0, L) > \frac{1}{w} \int_0^w \frac{xu'(x)}{wu'(w)} A(x) dx,$$

from Proposition 1. Furthermore, we have

$$\begin{aligned} \frac{d[xu'(x)]}{dx} &= xu''(x) + u'(x) \\ &= -u'(x)[R(x) - 1], \end{aligned}$$

and thus

$$\frac{d[xu'(x)]}{dx} \leq 0 \text{ if } R(x) \geq 1.$$

We deduce

$$\theta(0, L) > \frac{1}{w} \int_0^w A(x) dx \text{ if } R(x) \geq 1.$$

### **Proof of Proposition 2**

Using  $A' \leq 0$  in equation (17) allows us to write

$$\theta(0, L) \leq \frac{A(w-L)}{L^2 u'(w)} \int_{w-L}^w [x - (w-L)] u'(x) dx$$

Using  $R(x) \leq \bar{\gamma}$  and  $u''(x) < 0$  yields

$$\begin{aligned} \frac{d}{dx}[(x - (w - L))u'(x)] &= u'(x)\left[1 - R(x) - \frac{u''(x)}{u'(x)}(w - L)\right] \\ &\geq u'(x)[1 - R(x)] \\ &\geq u'(x)(1 - \bar{\gamma}) \\ &\geq u'(w)(1 - \bar{\gamma}), \end{aligned}$$

for all  $x \in [w - L, w]$ . Hence, we have

$$\begin{aligned} [x - (w - L)]u'(x) + (w - x)u'(w)(1 - \bar{\gamma}) &\leq [w - (w - L)]u'(w) \\ [x - (w - L)]u'(x) &\leq Lu'(w) + (w - x)u'(w)(\bar{\gamma} - 1) \\ &= u'(w)[L + (w - x)(\bar{\gamma} - 1)], \end{aligned}$$

for all  $x \in [w - L, w]$ . Consequently,

$$\begin{aligned} \theta(0, L) &\leq \frac{A(w - L)}{L^2 u'(w)} \int_{w-L}^w \{u'(w)[L + (w - x)(\bar{\gamma} - 1)]\} dx \\ &= \frac{A(w - L)}{L^2} \left[ \frac{L^2(\bar{\gamma} + 1)}{2} \right] \\ &= \frac{A(w - L)(\bar{\gamma} + 1)}{2}. \end{aligned}$$

Using  $C_p'' < 0$  and  $C(0, L) = 0$  allows us to write

$$\begin{aligned} C(p, L) &< C'(0, L)p \\ &= pL + \theta(0, L)pL^2 \\ &\leq pL \left[ 1 + \frac{A(w - L)(\bar{\gamma} + 1)L}{2} \right]. \end{aligned}$$

### Proof of Proposition 3

$T_\varepsilon(x) \equiv t(x, \varepsilon)$ , with  $\varepsilon > 0$ ,  $t(w - L, 0) = t'_x(w - L, 0) = t''_{xx}(w - L, 0) = 0$  and  $t'_x(x, 0) > 0$  for  $x > w - L$ . Let  $M > 0$ . Then, for small enough  $\varepsilon$ , there exist  $x_0(M, \varepsilon)$  and  $x_1(M, \varepsilon)$  such that

$$\begin{aligned} w - L &< x_0(M, \varepsilon) < x_1(M, \varepsilon), \\ T_\varepsilon(x_0(M, \varepsilon)) &= [x_0(M, \varepsilon) - (w - L)]^2 / L^2 M, \\ T_\varepsilon(x_1(M, \varepsilon)) &\leq [x_1(M, \varepsilon) - (w - L)]^2 / L^2 M, \\ T_\varepsilon(x) &< [x - (w - L)]^2 / L^2 M \text{ if } x_0(M, \varepsilon) < x < x_1(M, \varepsilon), \\ x_0(M, \varepsilon) &\longrightarrow w - L \text{ when } \varepsilon \longrightarrow 0, \\ x_1(M, \varepsilon) &\longrightarrow x_1^*(M) > 0 \text{ when } \varepsilon \longrightarrow 0. \end{aligned}$$

Thus, we have

$$T_\varepsilon(x) \leq \frac{[x_1(M, \varepsilon) - (w - L)][x - (w - L)]}{L^2 M},$$

or equivalently

$$A_\varepsilon(x) > \frac{L^2 M}{[x_1(M, \varepsilon) - (w - L)][x - (w - L)]},$$

if  $x_0(M, \varepsilon) < x < x_1(M, \varepsilon)$ . Hence, we may write

$$\begin{aligned} \theta(0, L) &> \frac{1}{2} \int_{w-L}^w k(x) A(x) dx \\ &> \frac{1}{2} \int_{x_0(M, \varepsilon)}^{x_1(M, \varepsilon)} \left( \frac{2[x - (w - L)]}{L^2} \times \frac{L^2 M}{[x_1(M, \varepsilon) - (w - L)][x - (w - L)]} \right) dx \\ &> \int_{x_0(M, \varepsilon)}^{x_1(M, \varepsilon)} \frac{M}{x_1(M, \varepsilon) - (w - L)} dx \\ &= M \times \frac{x_1(M, \varepsilon) - x_0(M, \varepsilon)}{x_1(M, \varepsilon) - (w - L)}. \end{aligned}$$

Since  $x_0(M, \varepsilon) \rightarrow w - L$  and  $x_1(M, \varepsilon) \rightarrow x_1^*(M)$  when  $\varepsilon \rightarrow 0$ , the right-hand side of the previous inequality goes to  $M$  when  $\varepsilon \rightarrow 0$ , and we deduce that  $\theta(0, L)$  is larger than  $M$  for small enough  $\varepsilon$ .

### Proof of Lemma 1

We have  $I^* > 0$  iff

$$\begin{aligned} \lambda &< \frac{u'(w - L) - u'(w)}{u'(w)} \\ &= -\frac{1}{u'(w)} \int_{w-L}^L u''(x) dx \\ &= \int_{w-L}^L A(x) \frac{u'(x)}{u'(w)} dx. \end{aligned}$$

Using  $Lk(x)/2 < 1$  for all  $x \in (w - L, w]$  gives

$$\int_{w-L}^L A(x) \frac{u'(x)}{u'(w)} dx > \frac{L}{2} \int_{w-L}^L k(x) A(x) \frac{u'(x)}{u'(w)} dx = L\theta(0, L),$$

and thus  $L\theta(0, L) \geq \lambda$  is a sufficient condition for  $I^* > 0$ .

### Proof of Proposition 5

The planner's program is to minimize  $\sum_i \alpha_i C_i$  under constraints (7), (8), (9) and (10). The Kuhn-Tucker multipliers associated with each set of constraints are respectively  $\gamma_i$ ,  $\phi_i(x_i)$ ,  $\eta$  and  $\rho$ . The optimality conditions are

$$\alpha_i - \gamma_i u'(w - C_i) = 0 \quad (18)$$

$$\gamma_i \pi q_i u'(w_{2i}(x_i)) f_i(x_i) - \eta(1 + \lambda) \alpha_i q_i f_i(x_i) + \phi_i(x_i) = 0, \quad (19)$$

$$u'(w_1) \sum_{i=1}^n (1 - \pi q_i) \gamma_i - \sum_{i=1}^n \int_0^{\bar{x}_i} \phi_i(x_i) dx_i - \rho + \eta(1 + \lambda) \sum_{i=1}^n \alpha_i q_i = 0, \quad (20)$$

$$-\eta + \rho c'_K(\pi, K) = 0, \quad (21)$$

$$\phi_i(x_i) \geq 0 \quad \text{and} \quad \phi_i(x_i) = 0 \quad \text{if} \quad w_{2i}(x_i) - w_1 + x_i > 0 \quad \forall i. \quad (22)$$

Let  $x_i$  be such that  $w_{2i}(x_i) - w_1 + x_i > 0$ . Thus, we have  $\phi_i(x_i) = 0$  from (22), and (19) gives

$$\pi \gamma_i u'(w_{2i}(x_i)) = \eta(1 + \lambda) \alpha_i. \quad (23)$$

(18) and (23) yield

$$u'(w_{2i}(x_i)) = \frac{\eta}{\pi} (1 + \lambda) u'(w - C_i). \quad (24)$$

Hence, if there exist  $x_i^0, x_i^1 \in [0, \bar{x}_i]$  such that  $w_{2i}(x_i^0) - w_1 + x_i^0 > 0$  and  $w_{2i}(x_i^1) - w_1 + x_i^1 > 0$ , then we must have

$$u'(w_{2i}(x_i^0)) = u'(w_{2i}(x_i^1)),$$

which implies

$$w_{2i}(x_i^0) = w_{2i}(x_i^1).$$

Consequently,  $w_{2i}(x_i)$  is constant over the set of  $x_i$  for which  $w_{2i}(x_i) - w_1 + x_i > 0$ , and we can write

$$w_{2i}(x_i) = w_1 - d_i,$$

with  $d_i < x_i$  for all  $x_i$  in this set, and from 24 we have

$$u'(w_1 - d_i) = \frac{\eta}{\pi} (1 + \lambda) u'(w - C_i). \quad (25)$$

Now let  $x_i$  be such that  $w_{2i}(x_i) - w_1 + x_i = 0$ . Using (18), (19) and (22) allows us to write

$$u'(w_{2i}(x_i)) = u'(w_1 - x_i) \leq \frac{\eta}{\pi} (1 + \lambda) u'(w - C_i).$$

Using (24), and  $u'' < 0$  we deduce  $x_i \leq d_i$ . Thus, we have established that there exists  $d_i$  such that

$$w_{2i}(x_i) = w_1 - d_i \quad \text{if} \quad x_i > d_i, \quad (26)$$

$$w_{2i}(x_i) = w_1 - x_i \quad \text{if} \quad x_i \leq d_i. \quad (27)$$

Let  $K \rightarrow K^*$  when  $\pi \rightarrow 0$  and  $c_0^* \equiv \lim_{\pi \rightarrow 0} c(\pi, K^*)$ . When  $\pi \rightarrow 0$ , we have  $w_1 \rightarrow w - c_0^*$  and  $C_i \rightarrow c_0^*$  from (10) and (7) respectively. (25) then gives  $d_i \rightarrow d^* \quad \forall i$  with

$$u'(w - d^*) = (1 + \lambda)u'(w - c_0^*) \lim_{\pi \rightarrow 0} \frac{\eta}{\pi}. \quad (28)$$

Using (18), (20), (21) and  $\sum_{i=1}^n \alpha_i = 1$  imply

$$\lim_{\pi \rightarrow 0} \left[ 1 - \frac{\eta}{c'_K(\pi, K^*)} + \eta(1 + \lambda) \sum_{i=1}^n \alpha_i q_i - \sum_{i=1}^n \int_0^{\bar{x}_i} \phi_i(x_i) dx_i \right] = 0. \quad (29)$$

Suppose that  $\eta$  does not go to zero when  $\pi$  does. In such a case, we would have  $\eta/c'_K(\pi, K^*) \rightarrow +\infty$  when  $\pi \rightarrow 0$  since  $c'_K(\pi, K^*) \rightarrow 0$ , and thus

$$\lim_{\pi \rightarrow 0} \left[ \eta \left[ \frac{1}{c'_K(\pi, K^*)} - (1 + \lambda) \sum_{i=1}^n \alpha_i q_i \right] \right] = +\infty.$$

Since  $\phi_i(x_i) \geq 0 \quad \forall i$ , this is in contradiction with (29). Thus, we have

$$\lim_{\pi \rightarrow 0} \left[ 1 - \frac{\eta}{c'_K(\pi, K^*)} - \sum_{i=1}^n \int_0^{\bar{x}_i} \phi_i(x_i) dx_i \right] = 0. \quad (30)$$

If  $d_i \leq 0$ , we have  $w_{2i}(x_i) - w_1 + x_i > 0$  and  $\phi_i(x_i) = 0 \quad \forall x_i > 0$ . Hence

$$\int_0^{\bar{x}_i} \phi_i(x_i) dx_i = 0.$$

If  $d_i > 0$ , we have  $\phi_i(x_i) = 0$  for  $x_i > d_i$ , and thus (18), (19) and (27) give

$$\int_0^{\bar{x}_i} \phi_i(x_i) dx_i = \int_0^{d_i} \phi_i(x_i) dx_i \quad (31)$$

$$= -\pi \alpha_i q_i \int_0^{d_i} \left[ \frac{u'(w - x_i)}{u'(w - C_i)} - \frac{\eta}{\pi} (1 + \lambda) \right] f_i(x_i) dx_i. \quad (32)$$

Using the fact that  $\eta \rightarrow 0$  when  $\pi \rightarrow 0$  gives

$$\lim_{\pi \rightarrow 0} \int_0^{\bar{x}_i} \phi_i(x_i) dx_i = 0,$$

and from (30) we derive

$$\lim_{\pi \rightarrow 0} \frac{\eta}{c'_K(\pi, K^*)} = 1.$$

Using (28) together with L'hôpital's rule, we finally deduce

$$\begin{aligned} u'(w - d^*) &= (1 + \lambda)u'(w - c_0^*)c''_{\pi K}(0, K^*) \\ &> u'(w), \end{aligned}$$

where the last inequality derives from  $\lambda > 0$  and  $c''_{\pi K}(0, K^*) \geq 1$ . Using  $u'' < 0$  gives  $d^* > 0$ . Since  $I_i(x_i) = w_{2i}(x_i) + x_i - w_1$ , we deduce that  $I_i(x_i) \rightarrow I^*(x_i) = \max(x_i - d^*, 0)$  when  $\pi \rightarrow 0$ .

### Proof of Proposition 6

Assume that  $\mathcal{A}$  minimizes  $\sum_{i=1}^n \alpha_i C_i$  in the set of feasible allocations, and suppose that it is not Pareto-optimal, then there exists a feasible allocation  $\hat{\mathcal{A}}$  and a group  $i_0$  such that  $\hat{C}_i - \hat{t}_i \leq C_i - t_i$  for all  $i$  and  $\hat{C}_{i_0} - \hat{t}_{i_0} < C_{i_0} - t_{i_0}$ . Consequently,

$$\sum_{i=1}^n \alpha_i (\hat{C}_i - \hat{t}_i) < \sum_{i=1}^n \alpha_i (C_i - t_i). \quad (33)$$

Since  $\mathcal{A}$  and  $\hat{\mathcal{A}}$  are feasible, we have

$$\sum_{i=1}^n \alpha_i t_i = \sum_{i=1}^n \alpha_i \hat{t}_i = 0, \quad (34)$$

and thus (33) and (34) give

$$\sum_{i=1}^n \alpha_i \hat{C}_i < \sum_{i=1}^n \alpha_i C_i,$$

which contradicts the fact that  $\mathcal{A}$  minimizes  $\sum_{i=1}^n \alpha_i C_i$  in the set of feasible allocations.

Conversely, assume that  $\mathcal{A}$  is a Pareto-optimal allocation, and suppose that it does not minimize  $\sum_{i=1}^n \alpha_i C_i$  in the set of feasible allocations. Thus there exists a feasible allocation  $\hat{\mathcal{A}}$  such that  $\sum_{i=1}^n \alpha_i \hat{C}_i < \sum_{i=1}^n \alpha_i C_i$ , and thus

$$\sum_{i=1}^n \alpha_i (\hat{C}_i - \hat{t}_i) < \sum_{i=1}^n \alpha_i (C_i - t_i). \quad (35)$$

Let us choose  $\hat{t}_i$  such that

$$\hat{t}_i = \hat{C}_i + t_i - C_i$$

for all  $i \neq i_0$ , which does not contradict the feasibility of  $\widehat{\mathcal{A}}$  if we choose

$$\widehat{t}_{i_0} = - \sum_{i \neq i_0} \widehat{t}_i. \quad (36)$$

We have

$$\widehat{C}_i - \widehat{t}_i = C_i - t_i \text{ for all } i \neq i_0. \quad (37)$$

Furthermore, (35),(36) and (37) give

$$\widehat{C}_{i_0} - \widehat{t}_{i_0} < C_{i_0} - t_{i_0}. \quad (38)$$

(37) and (38) contradict the fact that  $\mathcal{A}$  is Pareto-optimal.

## 6.4 Calibration of initial wealth and losses

INSEE, the French national statistical agency, provides an average estimated Gross National Product per capita of 32,227 euros<sup>34</sup> and an average age of 39.2 year old<sup>35</sup>. The French National Institute on Demographics (INED) provides an estimated life expectancy of 73.2 for the average 39.2<sup>36</sup> years old citizen. Lifetime wealth is obtained as the annual GDP per capita discounted at a 2% rate on a 34 year horizon. This yields an expected discounted future wealth of 805,310 euros. INSEE also provides an estimated average of 70,000 euros of current assets, which will be the financial loss that victims may incur. We therefore consider that initial wealth is 875,310 euros.

### 6.4.1 Group 1

The worst case scenario is a fatal outcome that occurs in states  $s_1 = 1$  and 2. As in Eeckhoudt et al. (2000) we assume that when this worst state materializes, the individual (in practice, her heir) is only able to retain a fraction, equal to  $\theta = 10\%$  of her initial wealth, that can be interpreted as a bequest parameter. In state  $s_1 = 2$ , the agent dies but does not suffer the financial loss. Direct losses in these catastrophic states are therefore equal to  $875,310(1 - \theta) = 787,780$  in state  $s_1 = 1$  and  $875,310(1 - \theta) - 70,000 = 717,780$  in state  $s_1 = 2$ . In state  $s_1 = 3$ , the agent suffers a severe health loss due to exposure to radioactivity, as well as a direct financial loss of all her financial assets. The cost of health treatment and the health induced reduction in future income is estimated in Eeckhoudt et al.(2000) at 260,000 euros. The direct loss in this state is therefore equal to 330,000 euros. In state  $s_1 = 4$ , the agent faces the 260,000 euros health loss and in state  $s_1 = 5$ , she faces the 70,000 euros financial loss.

Total losses are obtained by adding to the direct losses the indirect cost of the accident, assumed to be mutualized between all the agents who did not die. In the baseline scenario, the indirect loss is 1,440 euros per inhabitant.

### 6.4.2 Group 2

Agents in group 2 die in state  $s_2 = 1$ , face a severe disease in state  $s_2 = 2$  and a financial loss in state  $s_2 = 3$ . Their direct losses are therefore calibrated at 717,780 and 260,000 euros.

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<sup>34</sup><http://www.bdm.insee.fr>

<sup>35</sup><http://www.insee.fr/>

<sup>36</sup><http://www.ined.fr/>

Variable	Mean	Median	S.D.	Max	Min
Spread	0.0638	0.0525	0.0392	0.2000	0.0175
Expected loss	0.0235	0.0160	0.0232	0.1306	0.0001
Size (€million)	134.86	108.8984	113.6927	1128.8	17.9453

Table 6: Descriptive statistics for the 185 cat bonds

## 6.5 Descriptive statistics

Table 6 provides the summary statistics for the main variables. At 6.38%, the average spread is lower than in Braun (2015) who finds an average of 8.18% for the period 1997-2012. Average expected loss is very close to Braun (2015) (2.35% versus 2.08%) and the average value of capital issued (size) is higher in our data set (134.86 €million versus 97.34 €million), perhaps due to our inability to observe small private transactions.

## 6.6 OLS Estimates

Tables 7, 8, 9 and 10 provide the estimates of regression 12 for our fully specified model, by excluding the fixed cost and/or the risk premium among the explanatory variables. Expected loss, Risk premium and Fixed cost, respectively represent the terms  $\pi_i \mathbb{E}(\tilde{x})_i$ ,  $\pi_i [\mathbb{E}(\tilde{x}_i^2) - \pi(\mathbb{E}\tilde{x}_i)^2] K_i$ , and  $K_i^{-1}$ . 2017, Europe and Indemnity are the reference groups for the times dummies, the geographical area covered, and the trigger types, respectively. The coefficient estimates of Expected loss and Risk premium are positive and significant across the four specifications. Concerning the control variables, 2012 was a period of high prices, followed by a decline from 2013 to 2016. The geographical dummies point at the fact that cat bonds covering perils in the US are more expansive than in other countries. This is in accordance with Braun (2015). Finally, parametric triggers have a lower spread than indemnity triggers, which may be explained by the lower moral hazard entailed by parametric triggers.

The four regressions highlight the important role played by the risk premium term. We report, in the penultimate line of each table, the optimal level of coverage under scenario 1 and assumption  $\underline{R} = \overline{R} = 2$ . Without the risk premium term, the marginal cost of capital would be constant, hence the higher levels of coverage found under the specifications reported in Tables 9 and 10. On the other hand, the fixed cost term does not play a quantitatively important role. It is indeed divided among a large number of people, and therefore represents only a few cents per person. The last lines of each table report the premium paid under the same set of assumptions.

	coeff	t statistic	sign
Expected loss	1.4599	10.1094	***
Risk premium	0.0028	2.1819	**
Fixed cost	0.7490	1.6634	*
Fixed cost $\times$ 2011	3.3432	3.0260	***
Fixed cost $\times$ 2012	2.9848	7.5592	***
Fixed cost $\times$ 2013	1.1006	2.5057	**
Fixed cost $\times$ 2014	0.0040	0.0094	
Fixed cost $t \times$ 2015	0.0384	0.0877	
Fixed cost $\times$ 2016	-0.5876	-1.6232	*
Fixed cost $\times$ <i>Japan</i>	0.0926	0.3997	
Fixed cost $\times$ <i>US</i>	0.5149	1.8290	*
Fixed cost $\times$ <i>Other</i>	-0.0510	-0.1077	
Fixed cost $\times$ <i>Index</i>	-0.2880	-0.7308	
Fixed cost $\times$ <i>Param</i>	-1.1225	-3.3015	***
$R^2$	0.7860		
adjusted $R^2$	0.7698		
$K$ (billions)	0.7529		
$P$ (millions)	2.3125		

Table 7: Full specification

	coeff	t statistic	sign
Expected loss	1.4698	10.2101	***
Risk premium	0.0028	2.1814	**
Fixed cost $\times$ 2011	3.8580	3.5784	***
Fixed cost $\times$ 2012	3.4995	8.7510	***
Fixed cost $\times$ 2013	1.6547	4.5594	***
Fixed cost $\times$ 2014	0.6724	5.8811	***
Fixed cost $t \times$ 2015	0.5820	1.4407	
Fixed cost $\times$ 2016	-0.0396	-0.1135	
Fixed cost $\times$ <i>Japan</i>	0.1879	0.8498	
Fixed cost $\times$ <i>US</i>	0.6912	2.9144	***
Fixed cost $\times$ <i>Other</i>	0.2640	0.6896	
Fixed cost $\times$ <i>Index</i>	-0.2384	-0.5998	
Fixed cost $\times$ <i>Param</i>	-1.1137	-3.2829	***
$R^2$	0.7845		
adjusted $R^2$	0.7694		
$K$ (billions)	0.7554		
$P$ (millions)	1.5666		

Table 8: No fixed cost

	coeff	t statistic	sign
Expected loss	1.7862	14.7812	***
Fixed cost	0.4423	0.8028	
Fixed cost $\times$ 2011	3.6242	3.0851	***
Fixed cost $\times$ 2012	3.1972	6.8988	***
Fixed cost $\times$ 2013	1.3691	2.6737	**
Fixed cost $\times$ 2014	0.1394	0.2574	
Fixed cost $t \times$ 2015	0.2671	0.4961	
Fixed cost $\times$ 2016	-0.6864	-1.4555	
Fixed cost $\times$ <i>Japan</i>	0.2828	1.1127	
Fixed cost $\times$ <i>US</i>	0.5116	1.4984	
Fixed cost $\times$ <i>Other</i>	-0.0606	-0.1136	
Fixed cost $\times$ <i>Index</i>	-0.3867	-0.8408	
Fixed cost $\times$ <i>Param</i>	-1.2987	-3.5668	***
$R^2$	0.7223		
adjusted $R^2$	0.7029		
$K$ (billions)	1.9044		
$P$ (millions)	2.415		

Table 9: No risk premium

	coeff	t statistic	sign
Expected loss	1.7901	14.8286	***
Fixed cost×2011	3.9278	3.5102	***
Fixed cost×2012	3.5011	8.3361	***
Fixed cost×2013	1.6960	4.2504	***
Fixed cost×2014	0.5349	4.1758	***
Fixed cost $t \times 2015$	0.5881	1.1696	
Fixed cost×2016	-0.3608	-0.8465	
Fixed cost× <i>Japan</i>	0.3382	1.3638	
Fixed cost× <i>US</i>	0.6162	2.1877	**
Fixed cost× <i>Other</i>	0.1263	0.2949	
Fixed cost× <i>Index</i>	-0.3567	-0.7610	
Fixed cost× <i>Param</i>	-1.2924	-3.5505	***
$R^2$	0.7217		
adjusted $R^2$	0.7040		
K (billions)	1.9015		
P (millions)	1.9743		

Table 10: No risk premium/No fixed cost

## 6.7 Comparison with alternative models of cat bond pricing

This section compares our data set with Braun (2015)'s. In order to do so, we contrast the four cat bond pricing models estimated in Braun (2015), on a sample of 466 cat bond tranches covering a period from 1997 to 2012 (Table 11), with the same models estimated on our data set (Table 12). For comparison purposes, spreads are converted into basis points and expected losses are expressed in percentage points. Tables 11 and 12 display very similar estimates. All variables are significant, except  $\hat{\gamma}$  estimated in Lane's (2000) model, both with our own and Braun's (2015) data sets. The first model specifies spreads as a linear function of expected loss

$$s_i = \hat{\alpha} + \hat{\beta}\pi_i\mathbb{E}(\tilde{x})_i.$$

The second model has spread as a polynomial of the natural logarithm of the expected loss

$$s_i = \hat{\alpha} + \hat{\beta} \ln \pi_i \mathbb{E}(\tilde{x})_i + \hat{\gamma} [\ln \pi_i \mathbb{E}(\tilde{x})_i]^2.$$

The third model is from Lane (2000) and specifies

$$s_i = \pi_i \mathbb{E}(\tilde{x})_i + \hat{\alpha} \pi_i^{\hat{\beta}} \mathbb{E}(\tilde{x})_i^{\hat{\gamma}}.$$

Finally, Major and Kreps' (2002) model posits

$$s_i = \hat{\alpha} (\pi_i \mathbb{E}(\tilde{x})_i)^{\hat{\beta}}.$$

	Linear in EL coeff sign	Polynomial in ln(EL) coeff sign	Lane (2000) coeff sign	Major/Kreps (2002) coeff sign
$\hat{\alpha}$	307.6273 ***	423.5275 ***	411.8256 ***	470.2197 ***
$\hat{\beta}$	140.4200 ***	307.2772 ***	0.4130 ***	0.3610 ***
$\hat{\gamma}$		62.2991 ***	-0.0068	
adjusted $R^2$	0.6899	0.6817	0.5880	0.5564

Table 11: Artemis database

	Linear in EL coeff sign	Polynomial in ln(EL) coeff sign	Lane (2000) coeff sign	Major/Kreps (2002) coeff sign
$\hat{\alpha}$	312.70 ***	594.58 ***	415.74 ***	588.54 ***
$\hat{\beta}$	232.02 ***	399.49 ***	0.37 ***	0.43 ***
$\hat{\gamma}$		73.32 ***	0.08	
adjusted $R^2$	0.80	0.73	0.54	0.71

Table 12: Braun (2015) estimates

## 6.8 Robustness analysis

The following tables summarize the robustness analysis of our numerical results in section 4. Each table presents either optimal coverage or welfare gain for a given set of hypotheses. The cost of handling claims is set to  $\lambda = 0.3$ , which is viewed as a reasonable estimate in the literature. However, changes in this parameter have a very limited impact on the simulation results. The scenarios that are considered vary across lines. All results are expressed in euros. Within each table, we fix  $\bar{R}$  and let  $\underline{R}$  vary through the columns. From left to right, we therefore increase the agent's risk aversion. For each level of  $\bar{R}$  we provide two tables. The first delivers our estimates for the optimal level of coverage and the second computes the welfare gain relative to the no-coverage situation.

The most sensitive parameter is usually the subsistence level  $\theta$ . Our results indicate that, while the optimal coverage is robust to changes in  $\theta$ , the estimated welfare gains are quite sensitive. As expected, optimal coverage increases with the severity of the scenario under consideration and with the degree of risk aversion.

### 6.8.1 Optimal coverage and welfare gains with $\theta = 0.90$

$\underline{R}$	1	
Scenario	Cover	Welfare
1	0.2865	0.0101
2	0.3538	0.0128
3	0.3883	0.0141
4	0.4097	0.0147
5	0.4244	0.0150

Table 13: Coverage,  $\bar{R} = 1$

$\underline{R}$	1		2	
Scenario	Cover	Welfare	Cover	Welfare
1	0.6873	0.0556	0.7529	0.0786
2	0.9646	0.0814	1.1021	0.1202
3	1.1454	0.0957	1.3495	0.1456
4	1.2775	0.1038	1.5429	0.1617
5	1.3801	0.1083	1.7016	0.1719

Table 14: Coverage,  $\bar{R} = 2$

$\underline{R}$	1		2		3	
Scenario	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.0194	0.1474	1.1010	0.2313	1.1287	0.2816
2	1.5468	0.2197	1.7361	0.3405	1.8022	0.4063
3	1.9337	0.2592	2.2349	0.4001	2.3422	0.4725
4	2.2427	0.2813	2.6557	0.4348	2.8054	0.5107
5	2.5006	0.2933	3.0238	0.4553	3.2164	0.5337

Table 15: Coverage,  $\bar{R} = 3$

$\underline{R}$ Scenario	1		2		3		4	
	Cover	Welfare	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.2859	0.2826	1.3737	0.4602	1.4031	0.5605	1.4177	0.6224
2	2.0527	0.3996	2.2686	0.5998	2.3419	0.6940	2.3787	0.7467
3	2.6552	0.4568	3.0114	0.6612	3.1340	0.7489	3.1959	0.7957
4	3.1617	0.4864	3.6641	0.6925	3.8391	0.7762	3.9277	0.8198
5	3.6026	0.5014	4.2542	0.7092	4.4836	0.7909	4.6000	0.8328

Table 16: Coverage,  $\bar{R} = 4$

$\underline{R}$ Scenario	1		2		3		4		5	
	Cover	Welfare								
1	1.5004	0.4402	1.5895	0.6834	1.6189	0.7905	1.6335	0.8450	1.6422	0.8766
2	2.4813	0.5745	2.7081	0.7922	2.7838	0.8705	2.8215	0.9070	2.8441	0.9272
3	3.2866	0.6316	3.6696	0.8321	3.7984	0.8979	3.8629	0.9276	3.9015	0.9438
4	3.9861	0.6587	4.5357	0.8505	4.7220	0.9103	4.8154	0.9369	4.8714	0.9513
5	4.6116	0.6715	5.3346	0.8595	5.5814	0.9165	5.7054	0.9415	5.7798	0.9550

Table 17: Coverage,  $\bar{R} = 5$

### 6.8.2 Optimal coverage and welfare gains with $\theta = 0.975$

$\underline{R}$	1	
Scenario	Cover	Welfare
1	0.40609	0.0201
2	0.53913	0.0283
3	0.62009	0.0330
4	0.67669	0.0358
5	0.71916	0.0376

Table 18: Coverage,  $\bar{R} = 1$

$\underline{R}$	1		2	
Scenario	Cover	Welfare	Cover	Welfare
Scenario 1	0.9161	0.1430	0.9353	0.1655
Scenario 2	1.3933	0.2190	1.4368	0.2532
Scenario 3	1.7550	0.2649	1.8237	0.3062
Scenario 4	2.0530	0.2939	2.1472	0.3402
Scenario 5	2.3090	0.3125	2.4284	0.3624

Table 19: Coverage,  $\bar{R} = 2$

$\underline{R}$	1		2		3	
Scenario	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.3123	0.4423	1.3345	0.5298	1.3419	0.5664
2	2.1385	0.5830	2.1928	0.6681	2.2110	0.7016
3	2.8201	0.6468	2.9099	0.7273	2.9401	0.7584
4	3.4178	0.6804	3.5449	0.7579	3.5878	0.7879
5	3.9577	0.6993	4.1235	0.7753	4.1795	0.8048

Table 20: Coverage,  $\bar{R} = 3$

$\underline{R}$	1		2		3		4	
	Cover	Welfare	Cover	Welfare	Cover	Welfare	Cover	Welfare
Scenario 1	1.6146	0.7641	1.6375	0.8551	1.6451	0.8857	1.6489	0.9017
Scenario 2	2.7422	0.8519	2.8002	0.9138	2.8195	0.9337	2.8292	0.9444
Scenario 3	3.7149	0.8825	3.8130	0.9333	3.8457	0.9497	3.8621	0.9586
Scenario 4	4.5950	0.8965	4.7363	0.9423	4.7835	0.9572	4.8071	0.9655
Scenario 5	5.4104	0.9035	5.5970	0.9469	5.6595	0.9612	5.6908	0.9693

Table 21: Coverage,  $\bar{R} = 4$

$\underline{R}$	1		2		3		4		5	
	Cover	Welfare								
Scenario 1	1.8492	0.9243	1.8717	0.9676	1.8791	0.9787	1.8829	0.9840	1.8851	0.9874
Scenario 2	3.2280	0.9557	3.2864	0.9816	3.3058	0.9883	3.3154	0.9915	3.3212	0.9937
Scenario 3	4.4507	0.9655	4.5508	0.9860	4.5840	0.9913	4.6006	0.9939	4.6105	0.9957
Scenario 4	5.5785	0.9696	5.7239	0.9879	5.7723	0.9927	5.7964	0.9951	5.8109	0.9967
Scenario 5	6.6390	0.9716	6.8327	0.9888	6.8971	0.9934	6.9292	0.9958	6.9485	0.9972

Table 22: Coverage,  $\bar{R} = 5$

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