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Easily implementable time series forecasting techniques for resource provisioning in cloud computing

Michel Fliess$^{1,3}$, Cédric Join$^{2,3}$, Maria Bekcheva$^{4,5}$, Alireza Moradi$^4$, Hugues Mounier$^5$

Abstract—Workload predictions in cloud computing is obviously an important topic. Most of the existing publications employ various time series techniques, which might be difficult to implement. We suggest here another route, which has already been successfully used in financial engineering and photovoltaic energy. No mathematical modeling and machine learning procedures are needed. Our computer simulations via realistic data, which are quite convincing, show that a setting mixing algebraic estimation techniques and the daily seasonality behaves much better. An application to the computing resource allocation, via virtual machines, is sketched out.

Key words—Cloud computing, computing resources, virtual machines, forecasting, time series, nonstandard analysis, trend, quick fluctuation, seasonality, estimation, machine learning.

I. INTRODUCTION

The fast development of cloud computing (see, e.g., [6], [35]) is due to its tremendous benefits, which might be summarized by its pay-as-you-go (PAYG) feature. Providing adequate Quality-of-Service (QoS) and pricing necessitates obviously a good prediction of the workloads for a better resource provisioning. Several papers have already been written on this topic (see, e.g., [2], [4], [5], [8], [11], [24], [25], [27], [29], [30], [31], [32], [43], [48], [52], and the references therein). Various time series techniques prevail, especially those stemming from econometrics [37], [46], artificial neural networks and deep learning [33], [51], or a mixture of both [54]. They all need cumbersome mathematical modeling and/or machine learning procedures.

This communication suggests that another rather efficient approach to workload forecasting exists where the computer burden is much less demanding. This new viewpoint on time series, that was introduced ten years ago in financial engineering [18], [20], rests on a profound result [12] which was obtained via the language of nonstandard analysis [42].

It has been successfully employed for the prediction of traffic flow on highways [1] and mainly of photovoltaic energy [23]. It is worth mentioning that the signals corresponding to the workloads here and to the solar power in [23] are quite similar including their sampling.

Contrary to many publications,

- deterministic and probabilistic/statistical modelings become useless and therefore also parameter identification and/or machine learning,
- we are not trying to employ time series for taking into account quick oscillations of the workload. Realistic computer experiments [9] show that intelligent proportional controllers, or iPs, which are derived from the model-free control setting [19], achieve this elasticity property (see, e.g., [3], [14]) very well.

Lack of space prevents us to detail the computer implementations. Note however that this implementation of our algebraic tools has already been fruitfully completed not only with respect to time series (see, e.g., [23]), but also in control engineering and signal processing (see, e.g., [7], [9], [10], [41], [44], [45]).

Our paper is organized as follows. Our viewpoint on time series is developed in Section II, as well as three prediction techniques. Computer simulations are displayed and discussed in Section III. An application to the computing resource allocation, via virtual machines, is sketched out in Section IV. Section V presents some concluding remarks.

II. TIME SERIES

1) Nonstandard analysis: Nonstandard analysis was invented by Robinson [42] almost sixty years ago in order to give a rigorous definition of “infinitely small” and “infinitely large” numbers. For more readable initiations, see, e.g., [15], [16]. Let us emphasize that this unconventional achievement has also been employed in applied sciences (see, e.g., [26]).

2) A nonstandard definition of time series: Take the time interval $[0, 1]$. Introduce as often in nonstandard analysis the infinitesimal sampling

$$T = \{0 = t_0 < t_1 < \cdots < t_\nu = 1\}$$

where $t_{i+1} - t_i$, $0 \leq i < \nu$, is infinitesimal, i.e., “very small.” A time series $X$ is a function $T \rightarrow \mathbb{R}$.

Remark 1: In practice a time lapse of 1 minute should be viewed as quite small when compared to 1 day.

3) Quick fluctuations: A time series $X : T \rightarrow \mathbb{R}$ is said to be quickly fluctuating, or oscillating, around 0 [12], if, and only if, its integral $\int_a^b X dm$ [12] is infinitesimal for any appreciable interval $A$, i.e., an interval which is neither infinitesimally small nor infinitesimally large.

Remark 2: Let us emphasize that the probabilistic and/or statistical nature of those quick fluctuations do not play any role.

1LIX (CNRS, UMR 7161), École polytechnique, 91128 Palaiseau, France, Michel.Fliess@polytechnique.edu
2CRAN (CNRS, UMR 7039), Université de Lorraine, BP 239, 54506 Vandœuvre-lès-Nancy, France, cedric.join@univ-lorraine.fr
3AL.I.E.N. (Algèbre pour Identification & Estimation Numériques), 7 rue Maurice Barrès, 54330 Vézelise, France, (michel.fliess, cedric.join)@alien-sas.com
4Inagral, 128 rue de la Boëtie, 75008 Paris, France, (maria.alireza)@inagral.com
5Laboratoire des Signaux et Systèmes (L2S), Université Paris-Sud-CNRS-CentraleSupélec, Université Paris-Saclay, 91120 Gif-sur-Yvette, France, (maria.bekcheva, hugues.mounier)@l2s.centralesupelec.fr
4) The Cartier-Perrin theorem: The Cartier-Perrin theorem [12] states for a time series \( X : \mathbb{R} \rightarrow \mathbb{R} \) satisfying a rather weak integrability assumption the following additive decomposition

\[
X(t) = E(X)(t) + X_{\text{fluct}}(t)
\]

where
- the mean \( E(X)(t) \) is Lebesgue integrable,
- \( X_{\text{fluct}}(t) \) is quickly fluctuating.

The decomposition (1) is unique up to an additive infinitesimal quantity.

Remark 3: Replace the word “mean” by trend, which is perhaps more popular, in financial engineering for instance. Set therefore

\[
E(X)(t) = X_{\text{trend}}(t)
\]

Note however that the meaning of “trend” in the time series literature is most often quite different (see, e.g., [46]).

Remark 4: Our mean or trend should be understood as being quite close to the familiar notion of moving average [37], [46].

B. Three forecasting techniques

According to the Cartier-Perrin theorem (1) it only makes sense to forecast \( E(X)(t) \), i.e., the mean or the trend.

1) Scaled persistence and seasonality\(^2\): The persistence assumption reads in our context

\[
\tilde{X}_{\text{trend}}(t + h) = X_{\text{trend}}(t)
\]

where \( h > 0 \). Obviously Equation (2) will too often yield poor predictions. That is why we introduce scaled persistence via the most classic notion of seasonality in the time series literature [37], [46]. Our data, where the sampling period is equal to 1 minute, exhibit here a self-evident daily pattern. Equation (2) should then be replaced by

\[
\tilde{X}_{\text{trend}}(t + h) = \frac{X_{\text{trend}}(t - 1440 + h)}{X_{\text{trend}}(t - 1440)} X_{\text{trend}}(t)
\]

where
- \( 0 < h \leq 60 \text{min} \),
- \( \frac{X_{\text{trend}}(t - 1440 + h)}{X_{\text{trend}}(t - 1440)} \) is a correcting multiplicative term corresponding to the daily seasonality,
- \( 1440 = 60 \times 24 \) is the number of minutes during a single day.

2) Forecasting via algebraic estimation techniques\(^3\): Start with a polynomial time function

\[
p_1(t) = a_0 + a_1 t, \quad t \geq 0, \quad a_0, a_1 \in \mathbb{R},
\]

\(^1\)The presentation in [34] is less technical. We highly recommend this paper. It also includes a fruitful discussion on nonstandard analysis.

\(^2\)Persistence and scaled, or smart, persistence are quite often discussed elsewhere in the literature, essentially perhaps in meteorology (see, e.g., [49], [50]) and climatology (see, e.g., [38]).

\(^3\)For the estimation techniques, see also [22], [44], and [36] for more mathematical details.

of degree 1. Rewrite thanks to classic operational calculus (see, e.g., [53]) \( p_1 \) as

\[
P_1 = \frac{a_0}{s} + \frac{a_1}{s^2}
\]

Multiply both sides by \( s^2 \):

\[
s^2 P_1 = a_0 s + a_1
\]

Take the derivative of both sides with respect to \( s \), which corresponds in the time domain to the multiplication by \( -t \):

\[
s^2 \frac{dP_1}{ds} + 2s P_1 = a_0
\]

The coefficients \( a_0, a_1 \) are obtained via the triangular system of equations (4)-(5). We get rid of the time derivatives, i.e., of \( sP_1, s^2 P_1, \) and \( s^2 \frac{dP_1}{ds} \), by multiplying both sides of Equations (4)-(5) by \( s^{-n}, n \geq 2 \). The corresponding iterated time integrals are low pass filters. They attenuate the corrupting noises, which are viewed as highly fluctuating phenomena. A quite short time window is sufficient for obtaining accurate values of \( a_0, a_1 \).

The extension to polynomials of higher degree is obvious, and therefore also to truncated Taylor expansions.

Remark 5: In practice, the above integrals are of course replaced by straightforward linear digital filters.

Assume that the following rather weak assumption holds true: the mean \( E(X(t)) \) may be associated to a differentiable time function \([0, 1] \rightarrow \mathbb{R}\). Then, on a short time lapse, \( E(X(t)) \) is well approximated by a polynomial function of degree 1. The above calculations yield via sliding time windows numerical estimates \( \hat{X}_{\text{trend}}(t) \) and \( \hat{X}_{\text{trend}}(t) \) of the trend and of its derivative. Causality is taken into account via backward time calculations. In this setting ([18], [20]), forecasting the time series \( X(t) \) boils down to an extrapolation of its mean \( E(X(t)) \). If \( h > 0 \) is not too “large”, i.e., a few minutes in our context, a first order Taylor expansion yields the following estimate at time \( t + h \)

\[
\tilde{X}_{\text{trend}}(t + h) = X_{\text{trend}}(t) + \hat{X}_{\text{trend}}(t) h
\]

3) Algebraic estimation and seasonality: Replace Equation (6) by

\[
\tilde{X}_{\text{trend}}(t + h) = X_{\text{trend}}(t) + \hat{X}_{\text{trend}}(t - 1440 + h) h
\]

where \( \hat{X}_{\text{trend}}(t - 1440 + h) \) is the derivative 1 day backwards. With such a choice the derivative estimation needs not to be causal and becomes more precise and much easier to compute [36].

III. Computer simulations

A. Data

The time series \( X(t) \) was provided by the Company Inagral to whom two authors, M. Bekcheva and A. Moradi, belong:

- it was recorded during a time lapse \( \Delta \) equal to 10 days, with a sampling period of 1 minute,
- it represents the sum of processing times (CPU times) of the incoming user requests on a production Web Service, with a sampling rate of 1 minute.
Replace $\mathcal{X}(t)$ by (see Figure 1)

$$y(t) = \frac{\mathcal{X}(t)}{\max_{\tau \in \Delta} \mathcal{X}(\tau)}$$

Thus $0 \leq y(t) \leq 1$, $\forall t \in \Delta$. This normalization procedure hides any sensitive information. The practical meaning of our computer simulations should nevertheless remain clear.

**B. Comparison between the three different techniques**

Predictions stemming from the three Equations (3), (6), (7) are now compared. In order to derive a sound comparability procedure let us assume the following property: Consider the three time series associated to the three forecasting techniques. In each case the quick fluctuations around the trend may be viewed [17] as a noise like in engineering. It therefore yields a signal-to-noise ratio (SNR) [40]. The three corresponding SNRs are assumed to be approximatively equal.

Three time horizons are considered: 5, 30, 60 minutes. The results are reported in the table below, where the first (resp. second, third) column corresponds to Formula (3) (resp. (6), (7)):

<table>
<thead>
<tr>
<th>Prediction horizons</th>
<th>Pe [gain in %]</th>
<th>Al [gain in %]</th>
<th>Mi [gain in %]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t + 5$ min</td>
<td>15.12</td>
<td>12.47 [21.21%]</td>
<td>10.49 [44.17%]</td>
</tr>
<tr>
<td>$t + 30$ min</td>
<td>32.23</td>
<td>65.68 [-50.86%]</td>
<td>27.89 [15.56%]</td>
</tr>
<tr>
<td>$t + 60$ min</td>
<td>53.49</td>
<td>153.79 [-65.21%]</td>
<td>49.04 [9.08%]</td>
</tr>
</tbody>
</table>

The superiority of the approach from Section II-B.3, which is mixing the daily seasonality with our algebraic calculations, is indubitable. See Figure 1 for a view of the data and of the various trends according to the forecast horizons. Figure 2 (resp. 3, 4) displays forecasts according to Section II-B.1 (resp. II-B.2, II-B.3).

**IV. FORECASTING COMPUTING CAPACITY NEEDS FOR RESOURCE PROVISIONING**

Set $z(t) = 5 \times 10^6 y(t)$: it is the processing time in milliseconds defined in Section III-A. Lack of space compels to a single forecast horizon of 30 minutes via the single Formula (7). See Figure 5 for $\tilde{z}_{\text{trend}}(t + 30)$.

In our case, the QoS and the Web Service stability are ensured with 50% of processing usage on each single virtual CPU (vCPU) core. It yields the following predicted number of virtual machines:

$$\tilde{n}_{VM}(t + 30) = \frac{\tilde{z}_{\text{trend}}(t + 30)\text{min}}{30000}$$

where

- $VM$ is the well-known acronym of virtual machine.
- $30000$ is the number of milliseconds in a minute (in CPU time) for achieving the 50% use.

The computer results derived from Formula (8) are provided in Figure 6.

**Remark 6:** Being able to estimate the required amount of computing resources in advance, one can easily acquire the necessary resources via Cloud provider’s spare available capacity at a bargained price. For example, in the case of Amazon Web Services (AWS), the use of Spot Instances can reduce the compute price up to 90%.

**Remark 7:** As already stated in our Introduction, the unavoidable quick oscillations around $\tilde{n}_{VM}(t)$ and occasional uncertain load fluctuations are most efficiently taken into account [9] by iPs from model-free control.

**V. CONCLUSION**

We hope to have convinced the reader that easily implementable time series approaches yield convincing prediction results in cloud computing. If seasonality patterns are available, they should be exploited. They lead to notable simplifications and improvements.

This introductory communication should nevertheless be completed in several ways:

- Comparisons with other viewpoints on time series and, more generally, on forecasting should be investigated. Suitable metrics will be proposed.
- How would behave our setting with respect to different time lapses and samplings?
- The inevitable uncertainty of any forecasting technique plays obviously a critical role. It will be examined as in [23].

Let us conclude by mentioning that some works have suggested to use time series for detecting anomalies (see, e.g., [28], [47]) and enhancing privacy protection (see, e.g., [39], [55]). Our algebraic techniques [21], [23] might also be helpful there.

**REFERENCES**

Fig. 1: Time series and its trend

(a) View

(b) Zoom1

(c) Zoom2

(d) Zoom3

Fig. 2: Zoom1: Scaled persistence (—), Algebraic techniques (—) and Algebraic techniques & seasonality (—).

(a) Horizon: 5min

(b) Horizon: 30min

(c) Horizon: 60min


Fig. 3: Zoom2: Scaled persistence (---), Algebraic techniques (---) and Algebraic techniques & seasonality (---).

Fig. 4: Zoom3: Scaled persistence (---), Algebraic techniques (---) and Algebraic techniques & seasonality (---).

Fig. 5: Predicted workload via Section II-B.3: \( \hat{z}_{trend}(t + 30\text{min}) \)

Fig. 6: Predicted need of virtual machines via Section II-B.3: $\delta^{\nu}_{VM}(t+30min)$