# Active queue management for alleviating Internet congestion via a nonlinear differential equation with a variable delay

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**Abstract**

Active Queue Management (AQM) for mitigating Internet congestion has been addressed via various feedback control syntheses, especially P, PI, and PID regulators, by using a linear approximation where the "round trip time", i.e., the delay, is assumed to be constant. This constraint is lifted here by using a nonlinear modeling with a variable delay, introduced more than 20 years ago. This delay, intimately linked to the congestion phenomenon, may be viewed as a "flat output." All other system variables, especially the control variable, i.e., the packet loss ratio, are expressed as a function of the delay and its derivatives: they are frozen if the delay is kept constant. This flatness-like property, which demonstrates the mathematical discrepancy of the linear approximation adopted until today, yields also our control strategy in two steps: Firstly, designing an open-loop control, thanks to straightforward flatness-based control techniques, and secondly, closing the loop via Model-Free Control (MFC) in order to take into account severe model mismatches, like, here, the number of TCP sessions. Several convincing computer simulations, which are easily implementable, are presented and discussed.

## Highlights:

- In order to mitigate Internet congestion, this work is among the first ones to use a 20 years old modeling via a nonlinear differential equation with a variable delay, where a new flatness-like property is encountered: the delay is a flat output. Combining flatness-based open-loop control and closed-loop control via the intelligent proportional controller deduced from model-free control yields easily implementable and convincing computer experiments which display a remarkable robustness with respect to large uncertainties on the number of TCP connections.

- The above nonlinear modeling has mainly been employed until today to derive time-invariant linear delay approximate systems, which are quite popular, not only for investigating control-theoretic questions but also for computer experiments. The flatness-like property of the nonlinear model shows that freezing the delay implies that all other system variables, including the control one, are kept constant. The validity of the linear approximations is therefore questioned.

**Keywords:** Internet congestion, active queue management, flatness-based control, model-free control, intelligent proportional control, delay, nonstandard analysis, time series, prediction.

# 1 Introduction

In order to alleviate Internet congestion, an *active queue management* is a dropping packets policy inside a router buffer yielding a corresponding queue length management (see, e.g., [Adams(2013)], [Varma(2015)], [Grazia et al.(2017) ], [Hotchi(2021)] for surveys and comparisons). It is often related to various control techniques and should perhaps be viewed, according to [Varma(2015)], as "the largest human-made feedback-controlled system in the world." A modeling of the most popular *transmission control protocol* (*TCP*) has been derived more than twenty years ago in [Misra et al.(2000)] and [Hollot et al.(2002)] via some relationship with fluid mechanics. It is a nonlinear system of differential equations with a time-dependent delay, where the control variable is the packet loss ratio. Although this work is much cited, it seems, to the best of our knowledge, that almost only linear approximations with constant delays have been exploited to propose various applicable AQM techniques (see, however, [Barbera et al.(2010), Belamfedel Alaoui et al.(2018), Li and Peng(2022)]). Let us restrict our short review to a few examples where this approximation has been employed:

- The familiar *random early detection* (*RED*) algorithm, which was invented by [Floyd and Jacobson(1993)], has been commented by [Hollot et al.(2001)], [Ryu et al.(2005)].

- The well-known *proportional-integral enhanced* (PIE) controller is introduced by [Pan et al.(2013)].

- New algorithms are initiated by [Bisoy and Pattnaik(2017)], [Hotchi et al.(2020)], [Bisoy et al.(2021)], [Hotchi(2021)], [Hotchi and Kubo(2022)].

Our work relies on a remarkable attribute of the above mentioned nonlinear system: it should be called *flat* in a more or less analogous sense of [Fliess et al.(1995), Fliess et al.(1999)]. The delay is a *flat output*, the queue length another one. This means that the knowledge of the delay time variation, or of the queue length, determines all the other system variables, including the control one.

In particular, freezing the delay implies at once that all the other system variables are constant. This property thus questions the frequent use that seemed until today so self-evident, both for simulation and control purposes, of the linear approximations, where the delay is assumed to be constant and not the other system variables (see, e.g., [Alli-Oke(2022)], and references therein).

This paper shows that appropriate control-theoretic tools do exist for handling the nonlinear modeling:

- Open-loop control strategies are deduced at once by exploiting this flatness-like property of the nonlinear modeling. We here choose to regulate the delay:
  - It is obviously related to congestion.
  - *Controlling queue Delay* (*CoDel*) ([Nichols and Jacobson(2012)]), which has become a popular setting, also puts delay control on the forefront but via a completely different viewpoint.
- In order to counteract the unavoidable model mismatches (see, e.g., [Xu et al.(2015)] for a summary of the shortcomings of the above nonlinear modeling) and disturbances, we follow [Villagra and Herrero-Perez(2012)], [Fliess et al.(2021)], [Join et al.(2022a)] by closing the loop via *model-free* control in the sense of [Fliess and Join(2013), Fliess and Join(2022)]. The presence of a delay requires a predictor which cannot, here, be the celebrated Smith's predictor ([Smith(1957)]), because the latter is model-based (see, e.g., [Deng et al.(2022)] for a recent survey). We thus adapt here the viewpoint of [Hamiche et al.(2019)] for studying *supply chain management* (see also [Join et al.(2022b)]). This is achieved by removing the unpredictable *quick fluctuations* via a theorem due to [Cartier and Perrin(1995)] which is expressed in the language of *nonstandard analysis*. Note that this result has led to a new understanding of time series (see, e.g, [Fliess et al.(2018)], and references therein).

Our paper is organized as follows. After showing that the delay or the queue length can be viewed as a flat output, Section 2 explains the inherent weakness of linear approximations with a constant delay. Section 3 recalls the basic facts of model-free control, which has already been used successfully many times. In order to take the delay into account in the model-free approach, Section 4 exploits techniques stemming from nonstandard analysis. Numerical simulations are presented in Section 5: they show that the model mismatch on the number of TCP sessions is well compensated by the closed-loop control without any clear-cut superiority of the techniques developed in Section 4. Various questions are raised in Section 6.

# 2 Some consequences of the nonlinear modeling

## 2.1 Flatness

The nonlinear TCP/AQM network model ([Misra et al.(2000), Hollot et al.(2002)]) reads

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2[R(t-R(t))]}u(t-R(t)) \tag{1a}$$

$$\dot{Q}(t) = \frac{W(t)}{R(t)}N(t) - C(t) \tag{1b}$$

where

- $W(t) > 0$ is the length of the TCP window;
- $R(t) > 0$ is the *round trip time* ($RTT$) which appears as a time-dependent delay in Equation (1);
- $Q(t) > 0$ is the queue length;
- the control variable is the dropping packet policy $u(t)$, $0 \leqslant u(t) \leqslant 1$: it is called the *packet loss ratio* ([Alli-Oke(2022)]), or, as often in the literature, the *packet drop probability*;
- $C(t) > 0$ is the bottleneck link capacity;
- $N(t) > 0$ is the number of TCP sessions. It plays the role of external disturbance.

In practice $C(t)$ and $N(t)$ are piecewise constant. Thus $\dot{C}(t) = \dot{N}(t) = 0$, with the exception of a finite number of points on any finite-time interval. The RTT $R(t)$ and the queue length $Q(t)$ are related by a simple affine relation

$$R(t) = T + \frac{Q(t)}{C(t)} \tag{2}$$

where $T$ is the round trip propagation time. Assume, in the following computations, $\dot{C}(t) = \dot{N}(t) = 0$ on some time interval, Set $C(t) = C$, $N(t) = N$, where $C$ and $N$ are constant. Then Equation (2) yields $\dot{Q}(t) = C\dot{R}(t)$. Equation (1b) becomes

$$\dot{R}(t) = \frac{NW(t)}{CR(t)} - 1 \tag{3}$$

It yields

$$W(t) = \frac{CR(t)(\dot{R}(t)+1)}{N} \tag{4}$$

Thus Equations (4) and (1a) show that $W(t)$ and $u(t-R(t))$ depend on $R(t)$ and its first and second order derivatives. In other words, we may call System (1) *flat* and the RTT $R(t)$ a *flat output* (compare with [Mounier et al.(2003)]). Equation (2) shows that the queue length $Q(t)$ is another flat output.

**Remark 1** *Classic flatness has been formally defined in [Fliess et al.(1995)] via differential algebra and in [Fliess et al.(1999)] via differential geometry of infinite jets and prolongations. Combining differential and difference algebras (see, e.g., [Cohn(1970)]) permits a precise definition of flatness for nonlinear systems with constant delays ([Mounier and Rudolph(1998), Mounier and Rudolph(2008)]). Such a setting does not however work with a variable delay such as $R(t)$, since the time derivation $\frac{d}{dt}$ and the time shift $t \mapsto t - R(t)$ with a time-varying quantity do not commute. Before developing an adequate general mathematical formalism for our example, it may be wise to wait for other concrete case-studies. It would open a path to a new understanding of nonlinear systems with variable delays.*

## 2.2 Critical appraisal of the linear approximation

See [Alli-Oke(2022)] for a nice survey on linear approximations. Let $u_0$, $Q_0$, $W_0$, $R_0$ be the numerical values of $u(t)$, $Q(t)$, $W(t)$, $R(t)$ at an operating, or equilibrium, point. Contrarily to the variables $u$, $Q$, $W$, the delay $R$ is kept frozen at the value $R_0$: the delay in the linear approximation is constant. It is obvious that such an assumption contradicts Section 2.1, where Equations (2), (4) and (1a) show that $Q(t)$, $W(t)$ and $u(t)$ become constant when $R(t)$ is constant. This fact is casting some doubt not only about AQM via such approximations, but also on the computer simulations, which rely on it (see [Alli-Oke(2021), Alli-Oke(2022)], and the references therein).

**Remark 2** *Define the control variable $\delta u(t) = u(t) - u_0$ and the output variable $\delta Q(t) = Q(t) - Q_0$. They are often related in the literature (see,* *e.g., [Alli-Oke(2022)]) by the time-invariant linear delay system defined by the transfer function*

$$-\frac{(2N\frac{W_0}{2})^3 e^{-R_0 s}}{(R_0 s+1)(\frac{W_0 R_0}{2}s+1)}$$

*where the number $N$ of sessions is assumed to be constant. A system defined by such a transfer function is sometimes called* quasi-finite *([Fliess et al.(2002)]). The output $\delta Q$ is said to be* flat, *or* basic *([Fliess et al.(2002)]).*

# 3 Closed-loop control via model-free control without delay: a short review

## 3.1 Ultra-local model

Consider a single-input single-output (SISO) nonlinear system $(\Sigma)$. Denote by $\mathfrak{u}(t)$ (resp. $\mathfrak{y}(t)$) the control (resp. output) variable. It has been demonstrated ([Fliess and Join(2013)]) via elementary techniques from functional analysis and differential algebra that the often poorly known modeling of $(\Sigma)$ may be replaced, if some quite weak assumptions are satisfied, by an *ultra-local model*:

$$\mathfrak{y}^{(\nu)} = F + \alpha\mathfrak{u} \tag{5}$$

where $\alpha \in \mathbb{R}$ is chosen by the practitioner such that $\alpha\mathfrak{u}$ and $\mathfrak{y}^{(\nu)}$ are of the same order of magnitude: it does not need to be precisely known. Numerous successful applications (see, e.g., references in [Fliess and Join(2013), Fliess and Join(2022)]) have shown that $\nu = 1$ in Equation (5) yields most often a convenient ultra-local model:

$$\dot{\mathfrak{y}} = F + \alpha\mathfrak{u} \tag{6}$$

The following comments are useful:

- Equation (6) is only valid during a short time lapse: it must be continuously updated.
- $F$ is *data-driven*, i.e., it is estimated via the knowledge $\mathfrak{u}$ and $\mathfrak{y}$ ([Fliess and Join(2013)]):

$$F_{\text{est}}(t) = -\frac{6}{\tau^3}\int_{t-\tau}^{t} \left[(\tau - 2\sigma)\mathfrak{y}(\sigma) + \alpha\sigma(\tau-\sigma)\mathfrak{u}(\sigma)\right] d\sigma \tag{7}$$

The quantity $\tau > 0$ may be chosen to be quite "small." The above integral, which is a low pass filter, may, in practice, be replaced by a classic digital filter.

- $F$ subsumes not only the unknown structure of the system, which most of the time is nonlinear, but also any external disturbance.

### 3.2 Intelligent controllers and local stability

The loop is closed with the following *intelligent proportional controller* ([Fliess and Join(2013)]), or *iP*,

$$\mathfrak{u} = -\frac{F_{\text{est}} - \dot{\mathfrak{y}}^\star + K_P e}{\alpha} \tag{8}$$

where:

- $\mathfrak{y}^\star$ is the reference trajectory of the output,
- $e = \mathfrak{y} - \mathfrak{y}^\star$ is the tracking error,
- $K_P \in \mathbb{R}$ is a tuning gain.

Combining Equations (6) and (8) yields $\dot{e} + K_P e = F - F_{\text{est}}$. If the estimate $F_{\text{est}}$ is "good," i.e., if $F - F_{\text{est}} \approx 0$, then $\lim_{t \to +\infty} e(t) \approx 0$, if, and only if, $K_P > 0$.

## 4 Closed-loop control via model-free control with delay

### 4.1 Ultra-local model with delay

Set $v(t) = u(t - R(t))$ in Equation (1):

$$\begin{aligned}\dot{W}(t) &= \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2[R(t-R(t))]} v(t) \\ \dot{Q}(t) &= \frac{W(t)}{R(t)} N(t) - C(t)\end{aligned}$$

This trivial change of variable shows that the techniques from [Fliess and Join(2013)] remain valid for introducing the ultra-local model with a time-varying delay

$$\dot{\mathfrak{y}}(t) = \mathfrak{F} + \alpha \mathfrak{u}(t - R(t)) \tag{10}$$

where $\mathfrak{F}$ plays the same rôle as $F$ in Equation (6). Equation (7) becomes

$$\mathfrak{F}_{\text{est}}(t) = -\frac{6}{\tau^3} \int_{t-\tau}^{t} [(\tau - 2\sigma)\mathfrak{y}(\sigma) + \alpha\sigma(\tau - \sigma)\mathfrak{u}(\sigma - R(\sigma)] \, d\sigma \tag{11}$$

## 4.2 Prediction via time series

### 4.2.1 Time series and the Cartier-Perrin theorem

Consider the time interval $[0, 1] \subset \mathbb{R}$. Introduce as often in *nonstandard analysis* (see [Robinson(1974)], [Diener and Diener(1995)], [Lobry and Sari(2008)]) the infinitesimal sampling of $[0, 1]$: $\mathfrak{T} = \{0 = t_0 < t_1 < \cdots < t_\nu = 1\}$ where $t_{i+1} - t_i$, $0 \leqslant i < \nu$, is *infinitesimal*, *i.e.*, "very small". A *time series* $X(t)$ is a function $X : \mathfrak{T} \to \mathbb{R}$.

A time series $\mathcal{X} : \mathfrak{T} \to \mathbb{R}$ is said to be *quickly fluctuating*, or *oscillating*, if the integral $\int_A \mathcal{X} dm$ is infinitesimal, *i.e.*, very small, for any *appreciable* interval, *i.e.*, an interval which is neither "very small" nor "very large".

According to a theorem due to [Cartier and Perrin(1995)], the following additive decomposition holds for any time series $X$, which satisfies a weak integrability condition,

$$X(t) = E(X)(t) + X_{\text{fluctuation}}(t) \tag{12}$$

where

- the *mean*, or *trend*, $E(X)$ is "quite smooth";
- $X_{\text{fluctuation}}$ is quickly fluctuating.

The decomposition (12) is unique up to an additive infinitesimal: It means that the two terms on the right handside of Equation (12) are unique up to a "very small" additive quantitity.

### 4.2.2 Derivative estimate

Let us start with a polynomial time function of degree 1

$$p_1(\tau) = a_0 + a_1\tau$$

where $\tau \geqslant 0$, $a_0, a_1 \in \mathbb{R}$. Operational calculus (see, *e.g.*, [Yosida(1984)]) with respect to the variable $\tau$, permits to express $p_1$ as

$$P_1 = a_0/s + a_1/s^2$$

Multiply both sides by $s^2$:

$$s^2 P_1 = a_0 s + a_1 \tag{13}$$

Take the derivative of both sides with respect to $s$, which corresponds in the time domain to the multiplication by $-\tau$:

$$s^2 \frac{dP_1}{ds} + 2sP_1 = a_0 \tag{14}$$

The coefficients $a_0, a_1$ are obtained via the triangular system of linear equations (13)-(14). We get rid of the time derivatives, *i.e.*, of $sP_1$, $s^2P_1$, and $s^2\frac{dP_1}{ds}$, by multiplying both sides of Equations (13)-(14) by $s^{-n}$, $n \geq 2$. The corresponding iterated time integrals are low pass filters which attenuate the corrupting noises. A quite short time window is sufficient for obtaining accurate values of $a_0$, $a_1$.

**Remark 3** *See [Mboup et al.(2009)] and [Othmane et al.(2021), Othmane et al.(2022)] for more details. Note also that estimating derivatives via integrals seems to have been first introduced by [Lanczos(1956)].*

### 4.2.3 Prediction

Set the following forecast $X_{\text{forecast}}(t+\Delta T)$, where $\Delta T > 0$ is not too "large",

$$X_{\text{forecast}}(t + \Delta T) = E(X)(t) + \left[\frac{dE(X)(t)}{dt}\right]_e \Delta T \tag{15}$$

where $E(X)(t)$ and $\left[\frac{dE(X)(t)}{dt}\right]_e$ are estimated like $a_0$ and $a_1$ above. Let us stress that what we predict is the mean and not the quick fluctuations.

**Remark 4** *The above construction is obviously reminiscent of the* sliding window *techniques in the applied literature on time series (see, e.g., [Mélard(2008)]).*

Note that estimating $a_0$ and $a_1$ yields respectively the mean and the derivative.

#### 4.2.4 Local closed-loop stability

Equation (10) may be rewritten as

$$\dot{\mathfrak{y}}(t + S(t)) = \mathfrak{F}_{\text{forecast}}(t + S(t)) + \alpha\mathfrak{u}(t) \tag{16}$$

where the advance $S(t) > 0$ is defined by

$$S(t) = \min \{ \tau \mid \tau - R(\tau) = t \} - t \tag{17}$$

If $R(t)$ is "slowly" varying, it is clear that $R(t)$ and $S(t)$ remain close. Evaluating $S(t)$ however requires a prediction of $R(t)$. Replace therefore in Equation (17) $R(t)$ by the reference trajectory $R^\star(t)$. It yields

$$S^\star(t) = \min\{\tau | \tau - R^\star(\tau) = t\} - t \tag{18}$$

Equation (8) then becomes

$$\mathfrak{u}(t) = -\frac{\mathfrak{F}_{\text{forecast}}(t + S^\star(t)) - \dot{\mathfrak{y}}^\star(t + S^\star(t)) + K_P e(t + S^\star(t))}{\alpha} \tag{19}$$

where:

- the forecast of $\mathfrak{F}$ is obtained via Formulae (11) and (15);
- $e(t+S^\star(t)) = \mathfrak{y}_{\text{forecast}}(t+S^\star(t)) - \mathfrak{y}^\star(t+S^\star(t))$, where $\mathfrak{y}^\star$ is the reference trajectory and $e$ is the tracking error;
- $\mathfrak{y}_{\text{forecast}}(t + S^\star(t)) = \mathfrak{z}(t + S^\star(t))$ is obtained via the linear differential equation

$$\dot{\mathfrak{z}}(\tau) = \mathfrak{F}_{\text{forecast}}(\tau) + \alpha\mathfrak{u}(\tau - R^\star(\tau)) \quad t \leqslant \tau \leqslant t + S(t)$$

- $K_P \in \mathbb{R}$ is the tuning gain.

It yields

$$\dot{e}(t + S^\star(t)) + K_P e(t + S^\star(t)) = \mathfrak{F}(t + S^\star(t)) - \mathfrak{F}_{\text{forecast}}(t + S^\star(t)) \tag{20}$$

Local stability is ensured, i.e., $\lim_{t\to+\infty} \mathfrak{y}(t + S^\star(t)) \approx \mathfrak{y}_{\text{forecast}}(t + S^\star(t))$ , if

- $K_P > 0$,
- the forecast is "good," i.e., $\mathfrak{F}(t + S^\star(t)) - \mathfrak{F}_{\text{forecast}}(t + S^\star(t)) \approx 0$.

# 5 Computer simulations[1]

## 5.1 Various situations

Introduce the following control settings:

1. **Reference trajectory and nominal control**: The choice of a reference trajectory $R^\star(t)$ for the delay $R(t)$ yields at once via Section 2.1 an open-loop nominal control $u^\star(t)$ for $u(t)$, i.e.,

$$u^\star(t - R^\star(t)) = 2\left(\frac{1}{R^\star(t)} - \dot{W}^\star(t)\right)\left(\frac{R^\star(t - R^\star(t))}{W^\star(t)W^\star(t - R^\star(t))}\right)$$

   or

$$u^\star(t) = 2\left(\frac{1}{R^\star(t + S^\star(t))} - \dot{W}^\star(t + S^\star(t))\right)\left(\frac{R^\star(t)}{W^\star(t + S^\star(t))W^\star(t)}\right)$$

   and

$$W^\star(t) = \frac{C(t)R^\star(t)(\dot{R}^\star(t) + 1)}{N_0}$$

   where $N_0 = 60$ and $C = 3000$.

2. **Open-loop control (OL)**: Inject $u^\star$ in Equation (1) with $N(t) = N_0$.

3. **Closing the loop via an iP without delay (iP)**: In Equations (6), (7), (8), set $\mathfrak{y}(t) = e(t) = R(t) - R^\star(t)$, $\mathfrak{y}^\star(t) = 0$, $\mathfrak{u}(t) = \Delta u(t) = u(t) - u^\star(t)$. The iP (8) becomes

$$\Delta u = -\frac{F_{\text{est}} + K_P e}{\alpha}$$

   where $\alpha = -1000$, $K_P = 1$. Consider the estimation of $F_{\text{est}}$ via Formula (7) as a classic *finite impulse response* (*FIR*) (see, e.g., [Rabiner and Gold(1975)]). We thus apply a control $u(t)$ of the form

$$u(t) = u^\star(t) - \frac{F_{\text{est}} + K_P e}{\alpha}$$

   where the first part incorporates our knowledge of the system, and the second one deals with perturbations, model imperfections and unknown dynamics. It is clear that the practical implementation, which has already been achieved successfully a number of times, is straightforward.

[1]Contact C. Join (cedric.join@univ-lorraine.fr) for the simulation codes.

4. **Closing the loop via an iP with delay (iPWD)**: In Equations (10), (11), (19), set as above $\mathfrak{y}(t) = e(t) = R(t) - R^\star(t)$, $\mathfrak{y}^\star(t) = 0$, $\mathfrak{u}(t) = \Delta u(t) = u(t) - u^\star(t)$. The iP (19) becomes

$$\Delta u(t) = -\frac{\mathfrak{F}_{\text{forecast}}(t + S^\star(t)) + K_P e(t + S^\star(t))}{\alpha}$$

where, as above, $\alpha = -10$, $K_P = 1$. The calculations related to predictions are detailed in [Fliess et al.(2018)]. The sequel is similar to the case without delay.

## 5.2 Scenarios

We illustrate our control laws through two different scenarios, where the mismatch is the number of TCP sessions. The first scenario corresponds to normal operation: small variation of R. The second scenario represents an exit from a congestion situation and corresponds to a large variation of R. The command is designed with a known value of the number of connections N but operates with a N constant piecewise: in our simulation N first goes from 60 to 70 then goes down to 50. The number of connection N plays the role of an external disturbance. It is moreover this effect that the open loop curves Figs.1 and 4 show where the trajectory deviates from the reference one when the number of connections changes.

1. Scenario 1: $0.25\text{s} \leqslant R(t) \leqslant 0.3\text{s}$, $50 \leqslant N(t) \leqslant 70$.
2. Scenario 2: $0.3\text{s} \leqslant R(t) \leqslant 0.7\text{s}$, $50 \leqslant N(t) \leqslant 70$.

Table 1: Simulations and Figures

| Scenarios | OL | iP | iPWD |
|---|---|---|---|
| 1 | Figure 1 | Figure 2 | Figure 3 |
| 2 | Figure 4 | Figure 5 | Figure 6 |

Those Figures tell us that:

- the mismatch is well compensated by iPs with or without delay,
- iPs with or without delay exhibit very similar behaviors.

Our results are especially well displayed in Figure 7. Indeed, we can see that the behavior of the tracking errors are totally comparable. In other words the iP with delay seems to be useless!

# 6 Conclusion

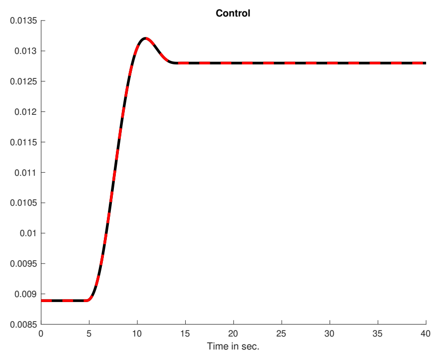
It has been shown that

- the linear constant delay approximations, which also play a key rôle in computer simulations, contradict the more complete nonlinear modeling;
- control-theoretic tools are available for an active queue management via this nonlinear modeling.

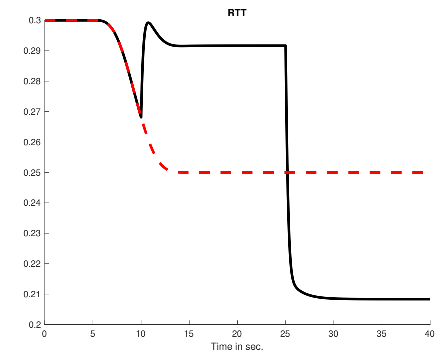
Many points remain of course to be addressed:

- Other mismatches and external disturbances ought to be examined: noisy measurements, abrupt changes of the round trip time, ... Would the *intelligent proportional-derivative controller* (*iPD*) advocated by [Fliess and Join(2022)] be helpful? See, e.g., [Sun et al.(2003)] and [Ryu et al.(2005)] for results with classic PD controllers.
- The simulations in Section 5.2 indicate the futility of an iP with delay in order to compensate a model mismatch. Without a precise mathematical analysis, it is not clear whether this property is always valid. Let us suggest nevertheless that the open-loop nonlinear control, where the delay is taken into account, is doing the job!
- The coefficient $\alpha$ in Equations (6) and (10), which does not need to be determined precisely, is obtained via trials and errors. A more subtle estimation would be welcome. Let us also add that important variations of some quantities like the number $N(t)$ of TCP connections might necessitate the introduction of a time-varying $\alpha$ (see [Gédoin et al.(2011), Moreno-Gonzales et al.(2022)] for first results in other engineering domains).
- Many network simulation for investigating Internet congestion (see, e.g., [Riley and Henderson(2010)], [Alli-Oke(2021), Alli-Oke(2022)], and references therein) seem to have employed time-invariant linear delay

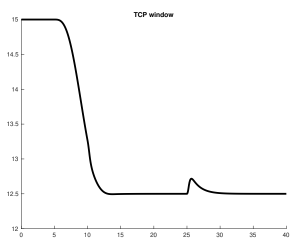
systems (see Section 2.2). It should therefore be most rewarding to develop and integrate the tools of this paper.



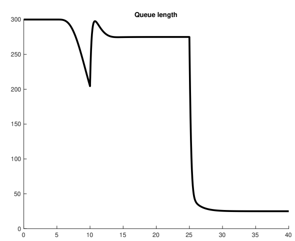

*(a)* Control (–) and nominal control (- -)



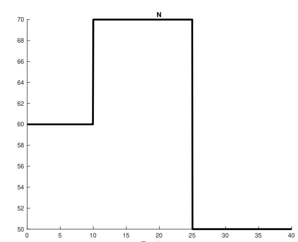

*(b)* $R$ (–) and reference trajectory (- -)




*(c)* TCP Window

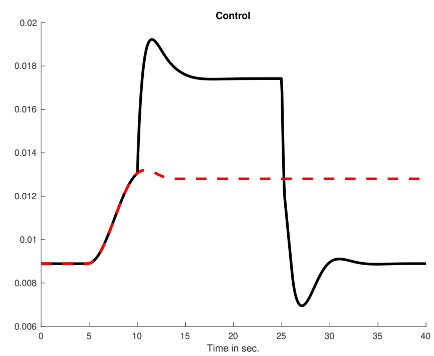



*(d)* Queue length

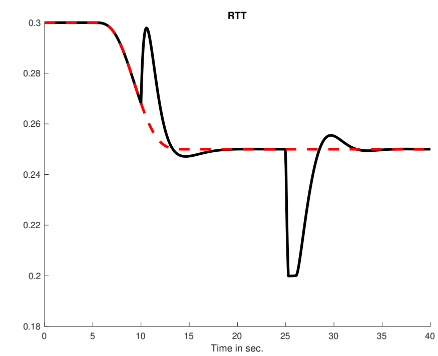



*(e)* Number of connections

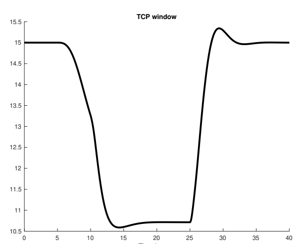
Figure 1: Scenario 1 – OL



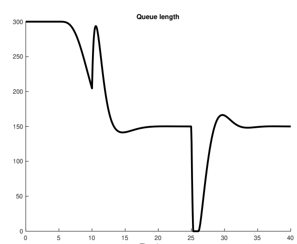

*(a)* Control (–) and nominal control (- -)



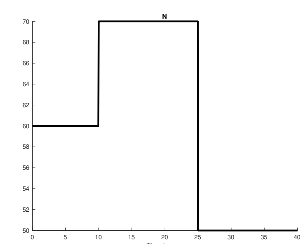

*(b)* $R$ (–) and reference trajectory (- -)




*(c)* TCP Window

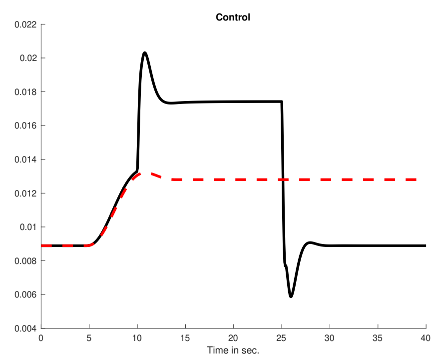



*(d)* Queue length

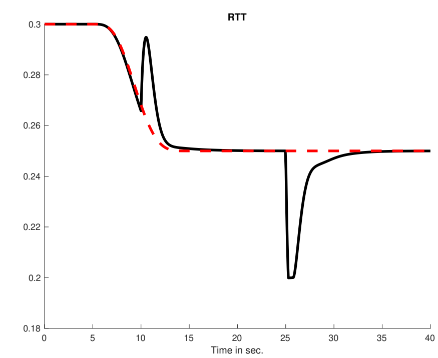



*(e)* Number of connections

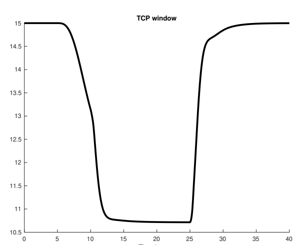
Figure 2: Scenario 1 – iP



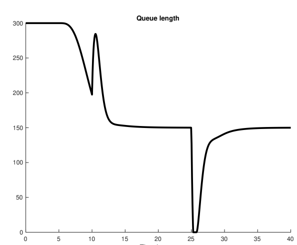

*(a)* Control (–) and nominal control (- -)



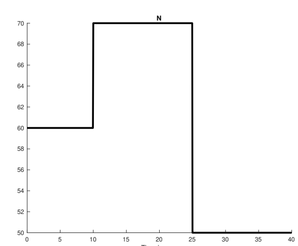

*(b)* $R$ (–) and reference trajectory (- -)




*(c)* TCP Window

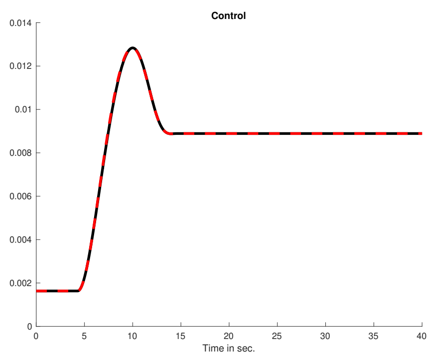



*(d)* Queue length

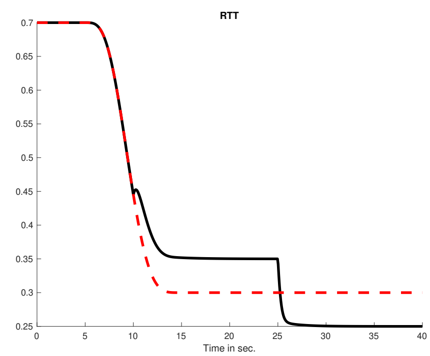



*(e)* Number of connections

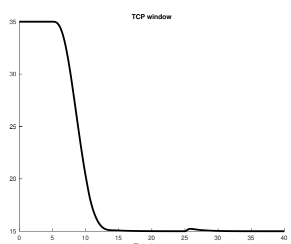
Figure 3: Scenario 1 – iPWD



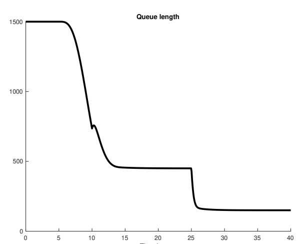

*(a)* Control (–) and nominal control (- -)



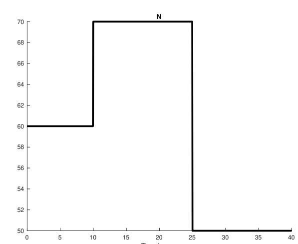

*(b)* $R$ (–) and reference trajectory (- -)




*(c)* TCP Window

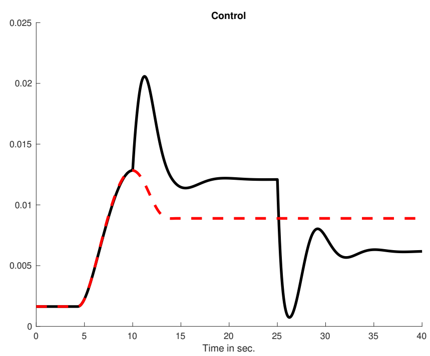



*(d)* Queue length

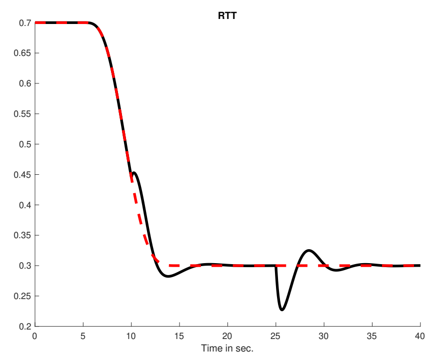



*(e)* Number of connections

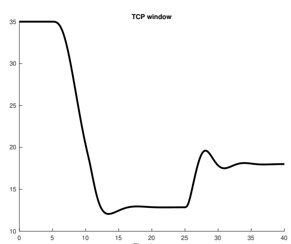
Figure 4: Scenario 2 – OL



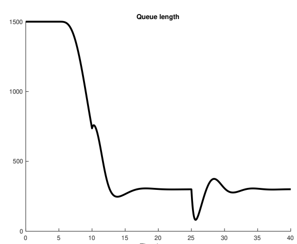

*(a)* Control (–) and nominal control (- -)



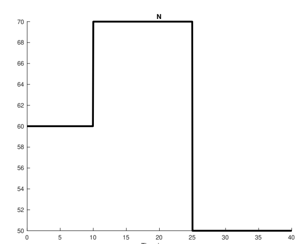

*(b)* $R$ (–) and reference trajectory (- -)




*(c)* TCP Window

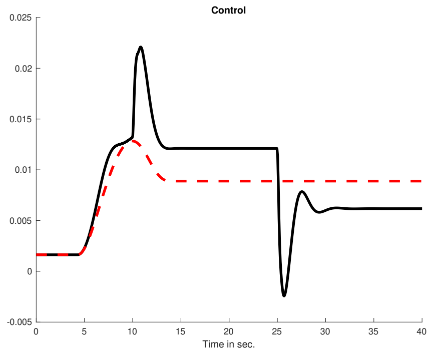



*(d)* Queue length

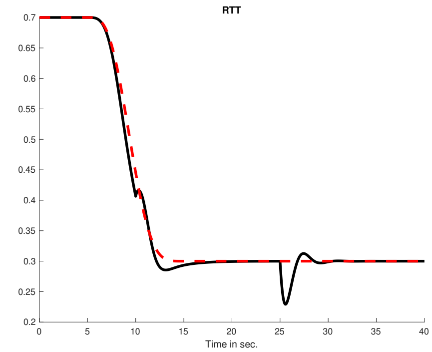



*(e)* Number of connections

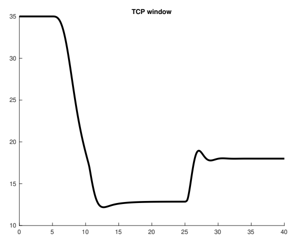
Figure 5: Scenario 2 – iP



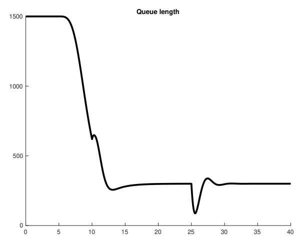

*(a)* Control (–) and nominal control (- -)



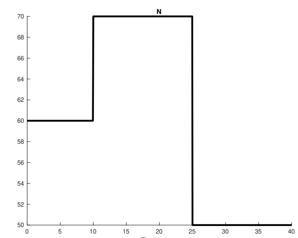

*(b)* $R$ (–) and reference trajectory (- -)




*(c)* TCP Window

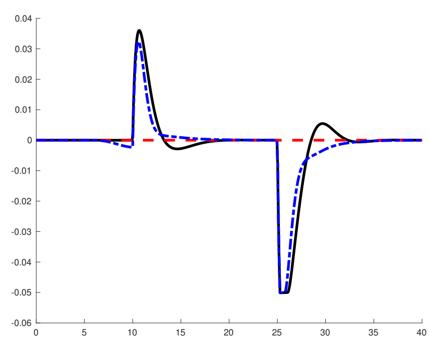



*(d)* Queue length

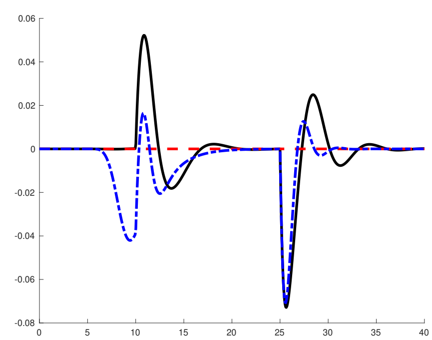



*(e)* Number of connections

Figure 6: Scenario 2 – iPWD




*(a)* Scenario 1 iP (–), iPWD (- .) and zero line (- -)




*(b)* Scenario 2 iP (–), iPWD (- .) and zero line (- -)

Figure 7: Tracking errors

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